LOAD–DISPLACEMENT ANALYSIS OF FOOTINGS IN DRY SAND

ANALYSE DE CHARGE–DEPLACEMENT DES PIEDS EN SABLE SECHE

ОПРЕДЕЛЕНИЕ ЗАВИСИМОСТИ НАГРУЗКА–ПЕРЕМЕЩЕНИЕ ДЛЯ СУХОГО ПЕСКА

A. GHAHRAMANI and A. SABZEVARI, Associate Professors, Pahlavi University, Shiraz (Iran)

SYNOPSIS. The authors present an analytical formulation for the load-displacement behavior of soils in a mixed boundary value problem concerning the strip footings on dry sand medium. This formulation leads to a better understanding of the soil stress-strain characteristics in the strip-footing problem at all working loads, rather than knowing just the soil bearing capacity at the failure as predicted conventionally. In the proposed incremental approach of this paper an iterative method is used for relating the velocity characteristic field to the stress field through the soil parameters derived from simple shear tests. Numerical computations are carried out for two types of loose and dense sand samples.

INTRODUCTION

In the recent advances on the influence of strains in soil mechanics, due to (Roscoe 1970), (James and Bransby 1970, 1971) and (Poorooshasb et al 1967), it is shown that basing the complete solution of the bearing capacity problem on the key assumption that the only required soil properties are those defining the state of failure is not quite correct. In fact, for a number of cases the experimental observations (James and Bransby 1970, 1971) have indicated gross deviations from the corresponding results obtained from the conventional stress analyses such as the Coulomb method (Harr 1966) or the method of Sokolovski (Sokolovski 1960, 1965).

It is indeed the aim of this Paper to revisit the bearing capacity problem by reformulating it as a mixed boundary value problem from the solution of which reliable load-displacement conditions in the soil mass can be calculated. In this theoretical formulation for solving such a boundary value problem the dry sand is assumed to behave as a rigid plastic continuum for which the constitutive parameters, the angle of internal friction ϕ and the angle of dilation ν, are deduced from the simple shear tests carried out at Cambridge University (Roscoe 1970) on loose as well as dense sand samples. Furthermore the direction of the major principal stress σ_1 is taken to coincide with the direction of the principal strain increment ϵ_1 (Roscoe and Poorooshasb 1963) and also the velocity characteristics are considered as the zero extension lines. Finally, the yielding is assumed to start on a K_0 line.

An incremental approach is employed in this work in which for each increment of the footing's rotation about one end an iterative method is used for relating the stress field to the corresponding velocity field. The stress field being predicted by the extended Sokolovski's limit analysis (Sabzevari and Ghahramani 1972) and the two fields being interconnected to each other by the constitutive parameters ϕ and ν.

ANALYSIS

An element of soil under limit equilibrium in a state of progressive failure is considered in Fig 1(a) in which the normal stresses are

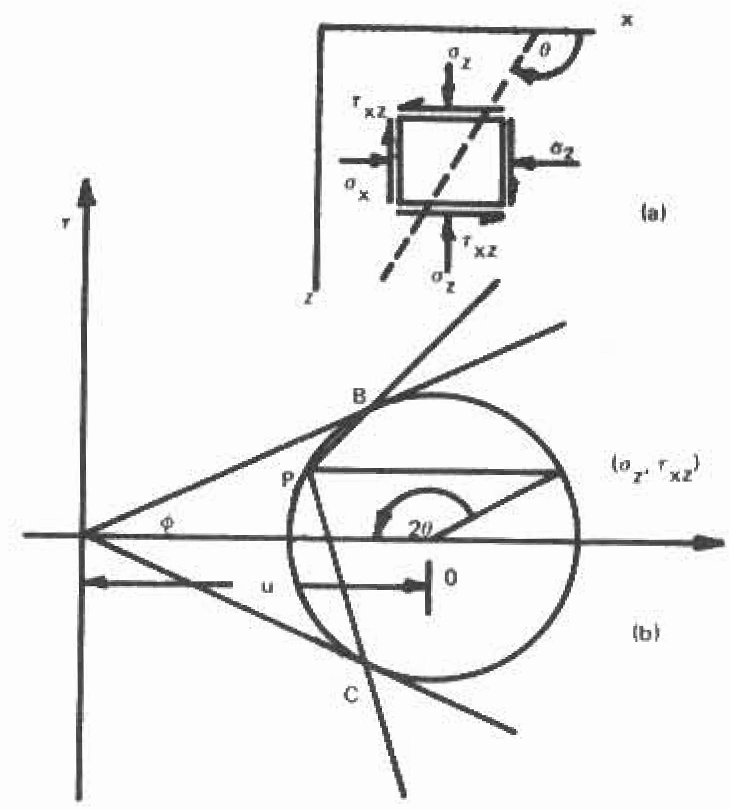

Fig. 1. A Soil Element Under Limit Equilibrium

lesignated by σ_x, σ_z, and τ_{xz} denotes the shear stress acting on he element. Hence the governing equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \tag{1}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma \tag{2}$$

n which γ is the soil unit weight. If ϕ denotes the molilized angle of internal friction then from the Mohr circle representation for he state of stress, Fig 1(b), the following relations are derived

$$\sigma_x = u(1 + \sin\phi\cos 2\theta) \tag{3}$$

$$\sigma_z = u(1 - \sin\phi\cos 2\theta) \tag{4}$$

$$\tau_{xz} = u\sin\phi\sin 2\theta \tag{5}$$

where $\phi = \phi(x, z)$ and u is the distance between the center of Mohr circle and the origin of $\sigma - \tau$ coordinates. Lines PB and PC in Fig. 1(b) are shown (Sabzevari and Ghahramani 1972) to be two characteristic directions on one of which the relations

$$\frac{dz}{dx} = \tan(\theta - \mu) \text{ and} \tag{6}$$

$$du - 2u\tan\phi d\theta = -(\gamma\tan\phi + u\frac{\partial\phi}{\partial z})dx + (\gamma + u\frac{\partial\phi}{\partial x})dz \tag{7}$$

hold, whereas along the second characteristic direction the following equations are shown to hold true,

$$\frac{dz}{dx} = \tan(\theta + \mu) \text{ and} \tag{8}$$

$$du + 2u\tan\phi d\theta = (\gamma\tan\phi + u\frac{\partial\phi}{\partial z})dx + (\gamma + u\frac{\partial\phi}{\partial x})dz \tag{9}$$

where μ is equal to $\frac{\pi}{4} - \frac{\phi}{2}$.

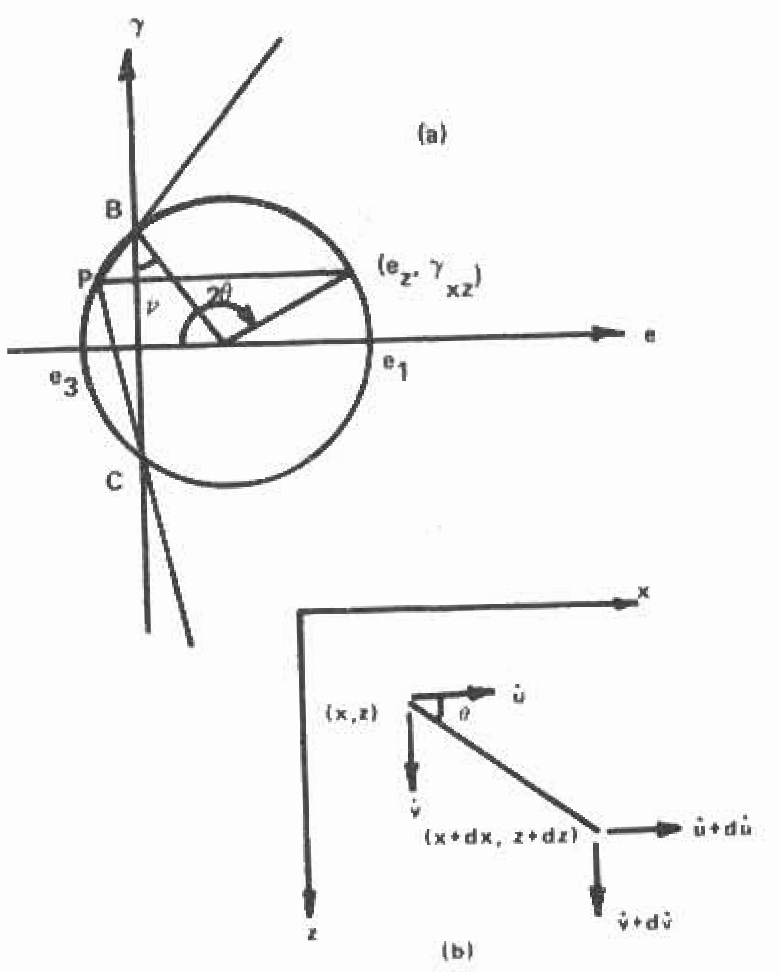

Fig. 2. (a) Mohr Circle Representation of Strain
(b) A Portion of Zero Extension Line

Equations (6) through (9) are then put in recurrence formulas from which the values of x, z, u and θ for the intersection point of two characteristic lines each one passing through a point with known values of x, z, u and θ can be calculated. The resulting recurrence formulas for stress together with the stress boundary conditions would yield the complete stress field.

Now the same element is let to undergo incremental displacement with $\dot{u}$ and $\dot{v}$ as its components in x and z directions respectively. The state of strain is presented by the Mohr circle of Fig 2(a) from which the angle of dilation ν is deduced to be

$$\nu = -\frac{\dot{e}_1 + \dot{e}_3}{\dot{e}_1 - \dot{e}_3} \tag{10}$$

where $\dot{e}_1$ and $\dot{e}_3$ are the principal incremental strains. Furthermore the direction of the principal incremental strain $\dot{e}_1$ and principal stress σ_1 coincide and make θ with the x axis. Since the strain rate along PB and PC are zero, then these are the zero extension lines and they coincide with the directions of two velocity characteristics (Poorooshasb et al 1967). The directions of these characteristics are respectively

$$\frac{dz}{dx} = \tan(\theta + a) \tag{11}$$

$$\frac{dz}{dx} = \tan(\theta - a) \tag{12}$$

where the angle a is equal to $\frac{\pi}{4} - \frac{\nu}{2}$.

A small portion of the zero extension line (behaving as a rigid link) is considered in Fig. 2(b) in which the incremental displacement components of one end are $\dot{u}$ and $\dot{v}$, and $\dot{u} + d\dot{u}$, $\dot{v} + d\dot{v}$ are those of the other end. Thus for the link element $(d\dot{u}/d\dot{v}) = -(dz/dx)$ and 653
consequently the following relations

$$\cos(\theta + a)d\dot{u} + \sin(\theta + a)d\dot{v} = 0 \tag{13}$$

$$\cos(\theta - a)d\dot{u} + \sin(\theta - a)d\dot{v} = 0 \tag{14}$$

are satisfied respectively on the appropriate characteristics. Hence similar to the case of stress, recurrence formulas for strain are obtained (Ghahramani and Sabzevari 1972). From the resulting recurrence relations together with the footings displacement rates on the boundary an incremental velocity field can be computed.

NUMERICAL COMPUTATIONS

Although the proposed method of analysis is quite general the numerical computations are carried out for two types of dry sand samples;

i) a loose sand sample for which

$$e_o = 0.75$$

$$\sin\phi = \frac{18\gamma + 0.4}{30\gamma + 1}$$

$$\sin\nu = \sin\phi - 0.58, \text{ and}$$

ii) a dense sand sample satisfying

$$e_o = 0.53$$

$$\sin\phi = 0.56 + \frac{4.61\gamma - 0.11}{193\gamma^2 + 1}$$

$$\sin\phi = 0.53 + 0.536 \sin\nu.$$

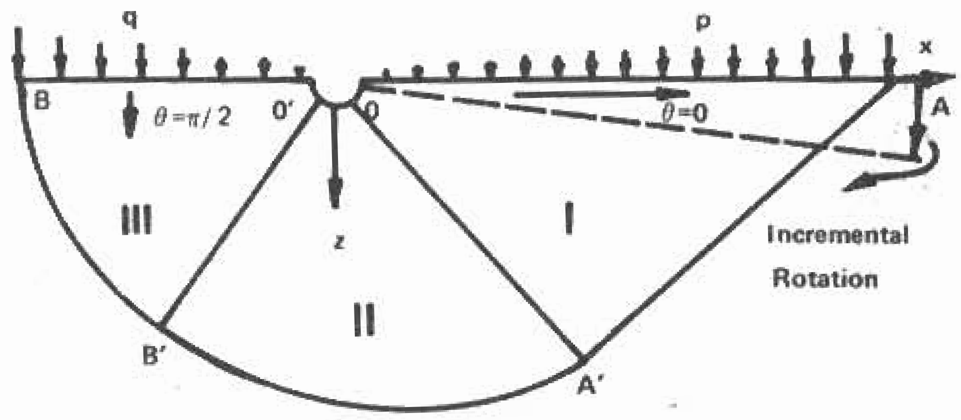

Fig. 3 The Strip-Footing Problem

The plastic equilibrium zone of an initially horizontal footing is shown in Fig. 3. The resulting plastic equilibrium region OAA'B'BO is composed of three distinct regions, i.e. OAA', OA'B'O' and O'B'B which respectively represent a Cauchy boundary value problem, a Goursat type problem and a mixed boundary value problem. Along the horizontal boundary OA, $z = 0$, $\theta = \pi/2$ and also the values of x and u are known. Hence by using the appropriate recurrence formulas together with known values of p on the boundary the complete stress field is computed (Ghahramani and Sabzevari 1972). In the computation of the first iteration of the stress field an initial value is assumed for the internal friction ϕ.

Once the first iterative stress characteristic field is constructed the corresponding velocity characteristics are obtained using the fact that the direction of the principal stress σ_1 and that of incremental principal strain $\dot{e}_1$ coincide. In addition, an initial value is also assumed for the angle of dilation ν.

The footing is now allowed to go through its first increment of rotation about one end. Thus knowing the incremental displacements on the boundary together with velocity characteristics we can construct the first iterative velocity field from which the strain field is also obtained. New values for ϕ and ν are now deduced from this strain field and then the aforementioned cycle is repeated until convergence is achieved. The final values of ϕ and ν for the first increment of the footings rotation are used as the initial values for the second increment of rotation.

In the aforementioned chain of computations the results of the simple shear tests carried out at Cambridge University (Roscoe 1970) are used for interconnecting the velocity field to the corresponding stress field. The incremental rotations of 2^o and 3^o are considered in this work for the dense and loose sand samples respectively.

RESULTS

The numerical calculations are performed by an IBM 1130-32 K computer for two sand samples, one being loose with e_o value of 0.75 and the other being a dense sand sample with its e_o equal to 0.53.

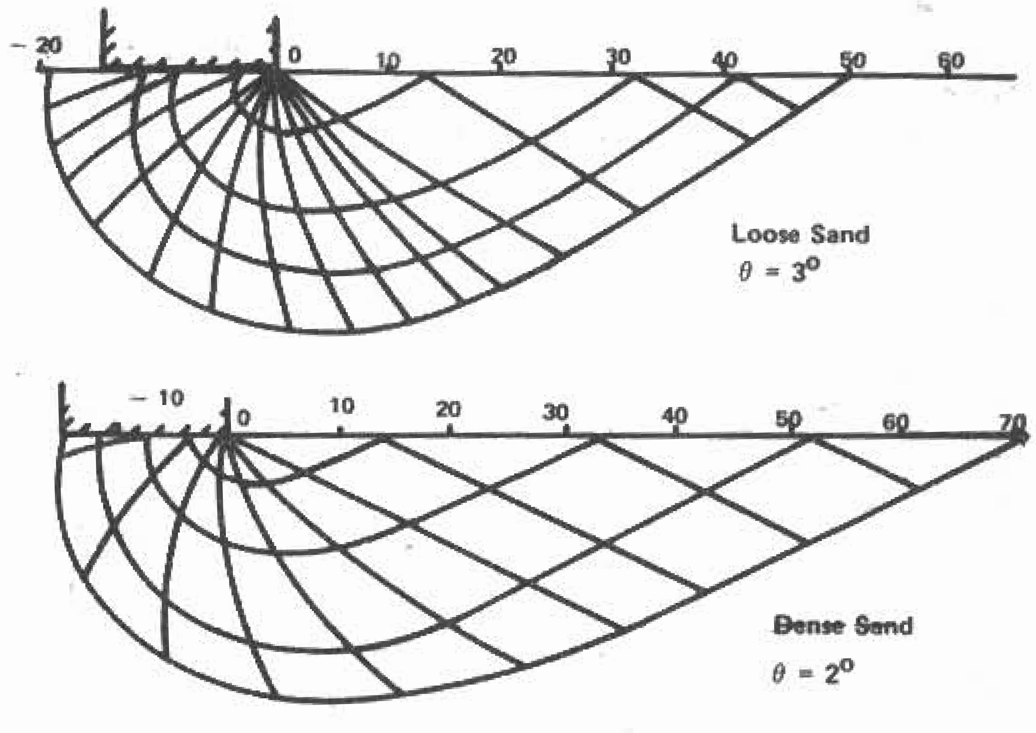

Fig. 4. Typical Stress Field

Typical stress and velocity field are presented in Figs. 4 and 5 for both dense and loose sand samples. As it is observed from Fig. 4 the stress characteristic field is more extended in the case of dense sand as compared to that of the loose sample. Such a behavior is explained by the fact that the angle of internal friction ϕ has larger values for dense sand. The velocity characteristic field, Fig. 5, also is more extended in the case of dense sand sample, and this time this is due to the fact that the dense sand here is basically expanding whereas the loose sample contracts.

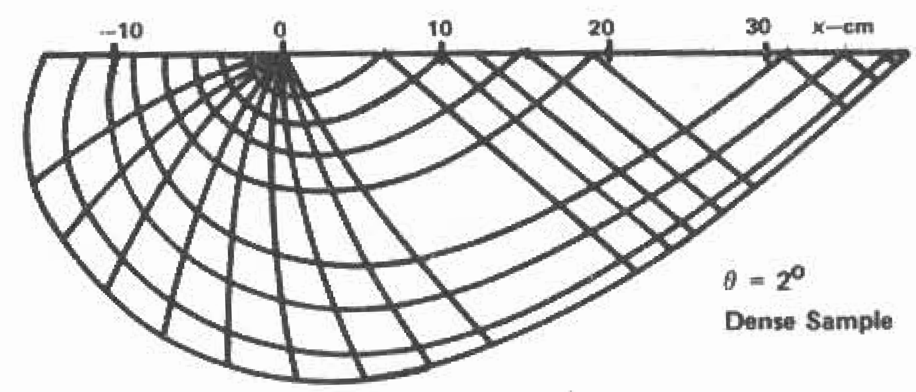

Fig. 5. Typical Velocity Field

Typical distributions of the normal as well as shear stresses acting on the footing are given by Fig. 6 in which the stresses are shown to increase progressively from one end, reaching maximum values at an intermediate point of the footing and then decreasing as the values of x get closer to the other end of the footing. Variations of the footing angle of internal friction δ are also plotted versus x on the same figure. It is possible to deduce certain nondimensional parameters such as force coefficient, moment coefficient and overall footing firction coefficient (Ghahramani and Sabzevari 1972) and then study the variations of these nondimensional coefficients with the footing's angle of rotation.

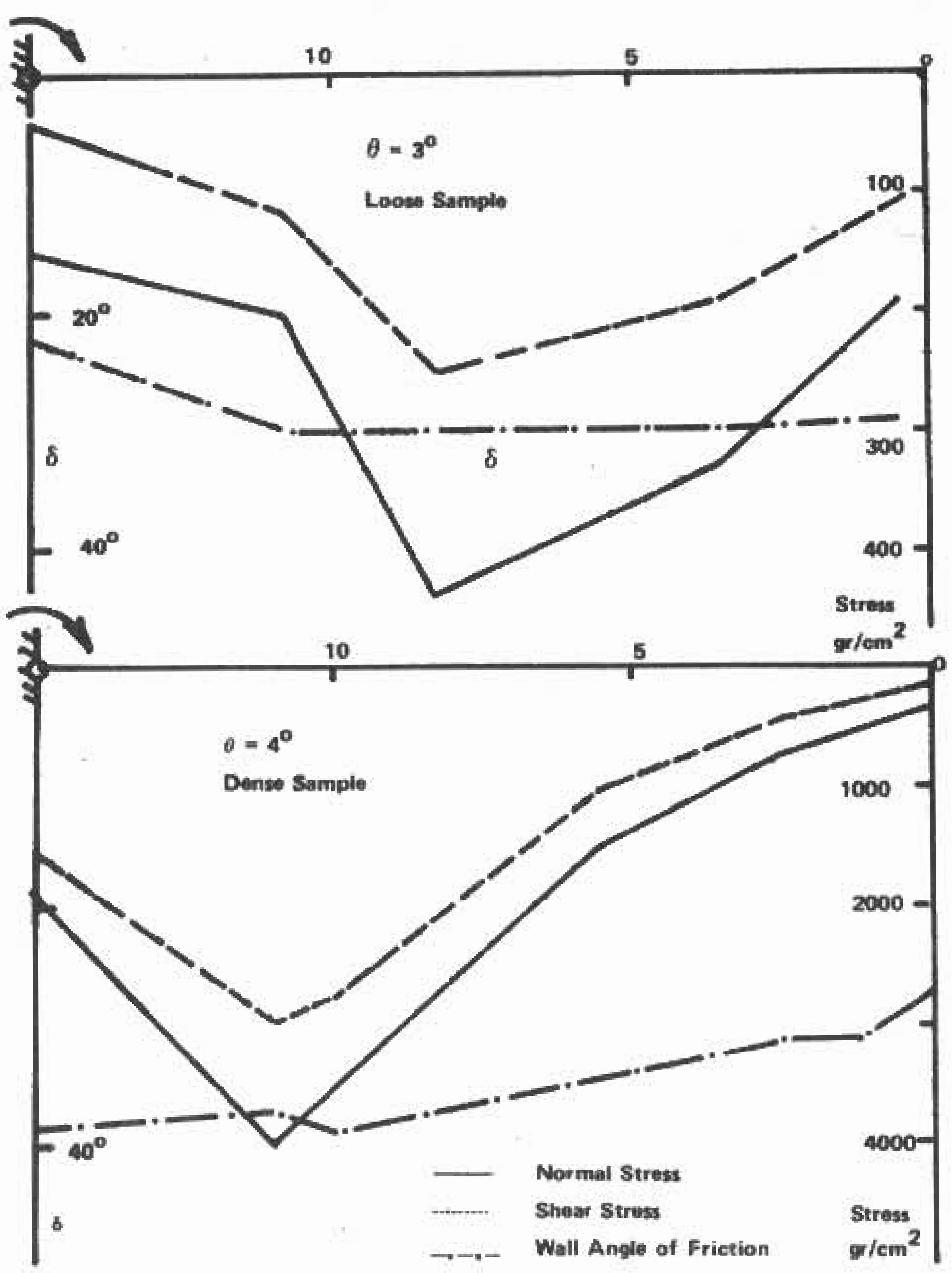

Fig. 6. Variations of Normal Stress, Shear Stress and Wall Angle of Friction

The induced displacements in the sand medium due to footing rotation are presented in Fig. 7 in each of which the initial surface OAB is moved to the new position OA'B. Heave of soil is noticed in this figure for both samples. Finally, distributions of shear strain γ along a ray beneath the footing is presented in Fig. 8.

CONCLUSIONS

The following conclusions are achieved in the theoretical studies of this paper;

1. It is proved that the strip footing problem admits itself to an analytical treatment in which there is no need for making an erroneous assumption, as in the conventional analysis that, of basing

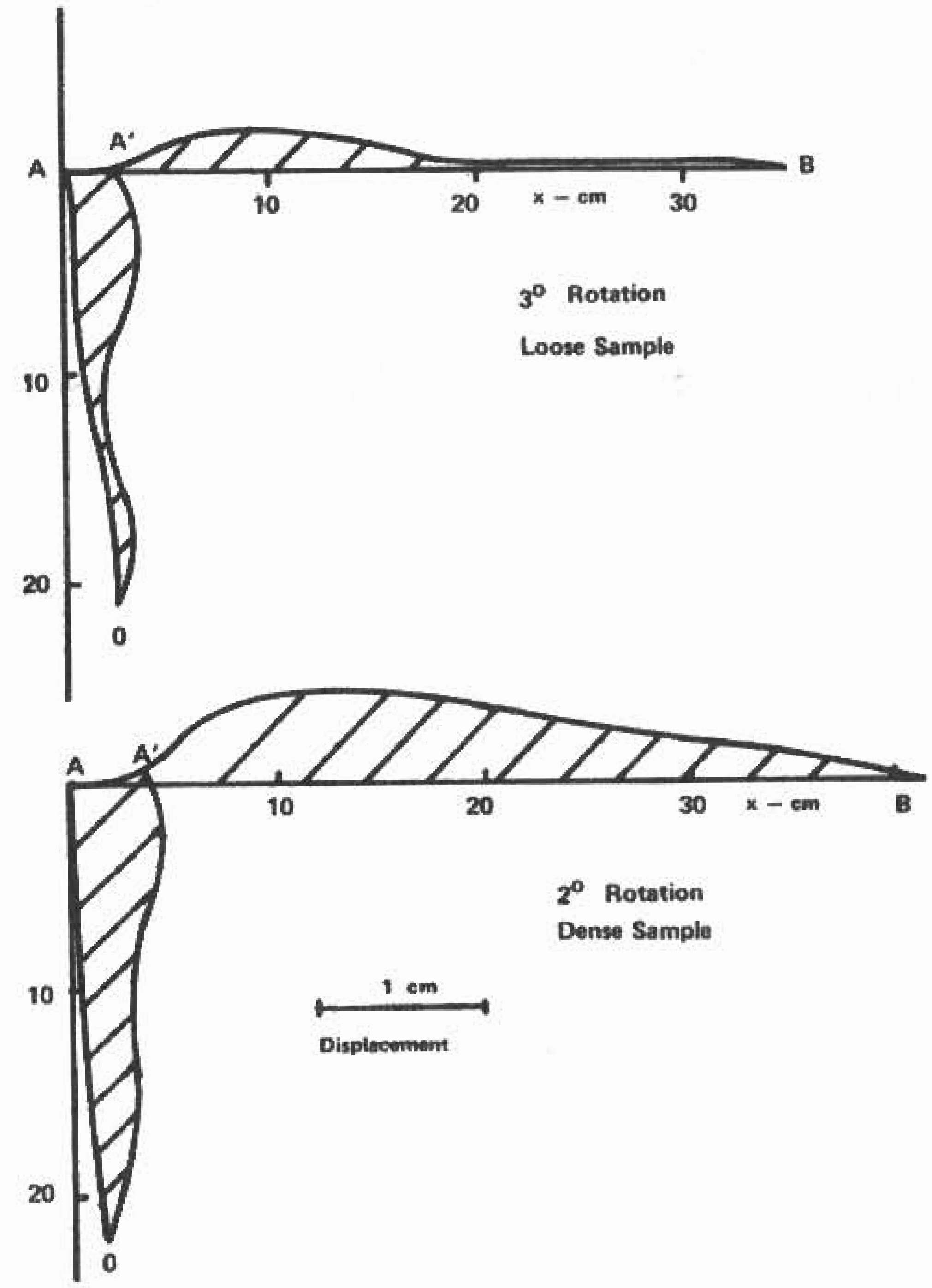

Fig. 7. Heave of the Soil

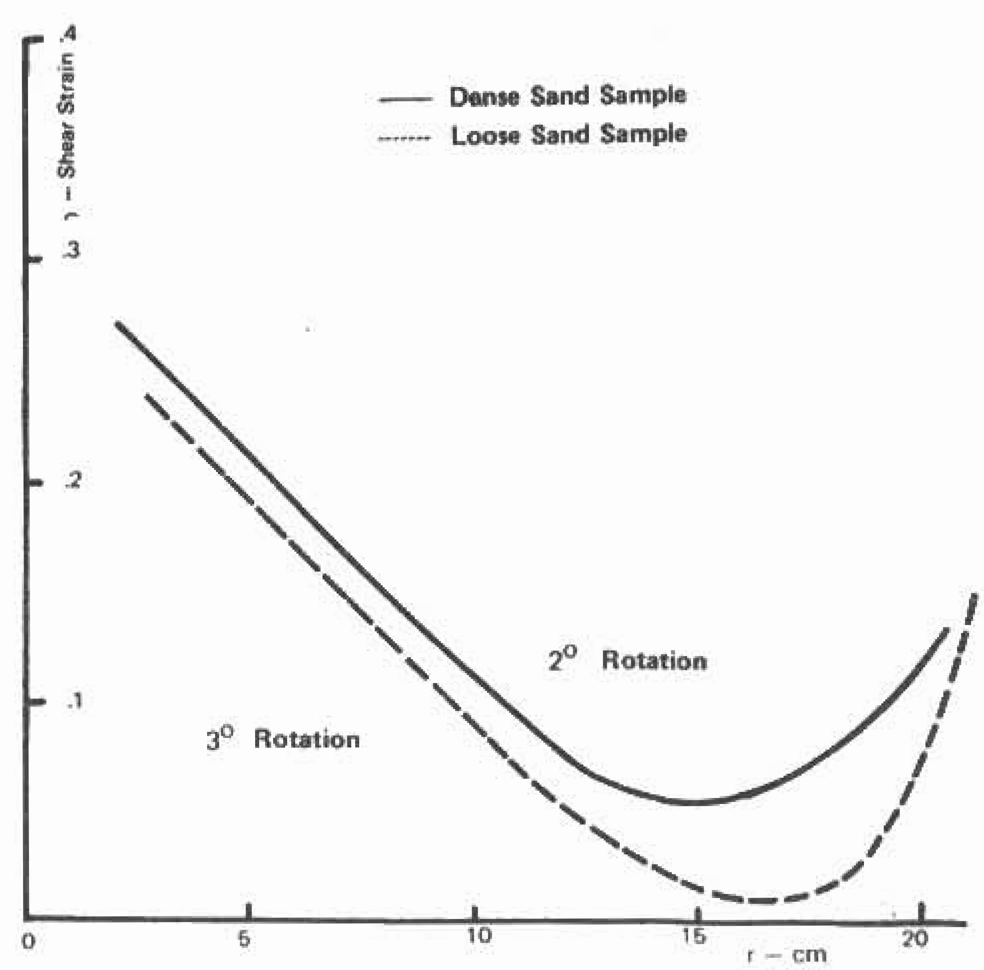

Fig. 8. Variations of Shear Strain

the complete solution on the soil properties at failure.

2. The iterative procedure of relating the strain field to stress field through the soil parameters derived from simple shear tests proved to be a fast converging one.

3. The perdicted values of normal and shear stresses are shown to vary along the footing and having maximum values at an intermediate point.

ACKNOWLEDGMENT

The authors wish to acknowledge the partial financial supports of the Iranian Geotechnique and Pahlavi University Research Council in funding this project. The authors are also thankful to Miss Rohan Kohanim for her kind assistance in preparing the manuscript.

REFERENCES

GHAHRAMANI, A., and SABZEVARI, A. (1972), "A Load-Displacement Analysis for Passive Earth Pressure Problem", submitted for publication to Geotechnique.

HARR, M. E., (1966), "Foundation of Theoretical Soil Mechanics" McGraw-Hill, New York.

JAMES, R. G., and BRANSBY, P. L., (1970), "Experimental and Theoretical Investigation of a Passive Earth Pressure Problem", Geotechnique Vol. 20, No. 1, 17-37.

JAMES, R. G., and BRANSBY, P. L., (1971), "A Velocity Field for Some Passive Earth Pressure Problems", Geotechnique, Vol. 21, No. 1, 61-83.

POOROOSHASB, H. B., HOLUBEC, I. and SHERBOURNE, A. N. (1967), "The Yielding and Flow of Sand in Triaxial Compression", Can. Geotech. J., Vol. 3, No. 4., 179-190 and Vol. 4, No. 4, 376-397.

ROSCOE, K. H., (1970), "The Influence of Strains in Soil Mechanics", Geotechnique, Vol. 20, No. 2, 129-170.

ROSCOE, K. H., and POOROOSHASB, H. B. (1963), "A Theoretical and Experimental Study of Strains in Triaxial Compression on Normally Consolidated Clays", Geotechnique, Vol, 13, No. 1, 12-38.

SABZEVARI, A. and GHAHRAMANI, A., "The Limit Equilibrium of Bearing Capacity and Earth Pressure Problems in Nonhomogeneous Soils", (1972), accepted for publication in Soils and Foundations.

SOKOLOVSKI, V. V., (1960), "Statics of Soil Media", Butterworth, London.

SOKOLOVSKI, V. V., (1965), "Statics of Granular Media", Pergamon Press, Oxford.