THE LIMIT EQUILIBRIUM ANALYSIS OF BEARING CAPACITY AND EARTH PRESSURE PROBLEMS IN NONHOMOGENEOUS SOILS

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ABSTRACT

An analytical study concerning the limit equilibrium of non-homogeneous soil medium satisfying nonlinear yield criterion is presented. The method of characteristics is applied to derive the recurrence formulas and consequently the slip line field.

This formulation is then applied to the problems of bearing capacity and earth pressure for which the slip line fields as well as the normal stress distributions are obtained for a special case of nonlinearity of the failure function.

Furthermore, the results obtained in this work for the bearing capacity and earth pressure problems are compared with those predicted by the conventional theories of homogeneous soils.

Key words: bearing capacity, earth pressure, inhomogeneity, nonlinear, limit analysis

INTRODUCTION

The problem of limit equilibrium of a soil mass has been solved by Sokolovsky (8), Harr (5) and others (3, 4) employing the Coulomb yield function \( \tau = c + \sigma \tan \phi \), where \( c \) denotes the cohesion, \( \phi \) is the angle of internal friction of the medium and the terms \( \tau \) and \( \sigma \) are the shear and normal stresses respectively. These investigators have used the method of characteristics to find the slip line field for specific boundary value problems in which the stresses applied on the boundary are known.

In the limit equilibrium analysis of Sokolovsky and others for the uniform Coulomb soil medium, the two equations of equilibrium (for two dimensional case) are satisfied, and furthermore the state of stress is such that its Mohr circle just touches the straight yield line \( \tau = c + \sigma \tan \phi \).

The same method has been successfully employed by Terzaghi (9), Meyerhof (6) and Biarez, Burel and Wack (3) for obtaining solutions to the problems of bearing capacity and earth pressure in homogeneous soil media. Furthermore, Sokolovsky's method has been recently extended to the case of soils exhibiting nonlinear failure criterion (1).

However, it is known that soils in reality are rarely uniform and furthermore the inhomogeneity of the medium could have an important effect on the slip line fields as well as the stress distribution. In fact Roscoe and Poorooshab (7) have demonstrated the significant role of soils inhomogeneity on the determination of strain fields in the yielding soil.

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Written discussions on this paper should be submitted before July 1, 1973.
medium.

It is therefore the aim of this work to present the limit equilibrium analysis of a soil medium having a general yield function which varies with $\sigma$, $x$ and $z$. In this formulation the method of Sokolovsky is generalized and the method of characteristics is used to obtain the slip line field. Finally the problem is solved by the finite difference.

The generalized theory is then used to obtain analytical solutions for the bearing capacity problem and the problem of earth pressure in nonhomogeneous soil mass having a general failure criterion. Furthermore, by means of a number of numerical examples, the deviations in slip line fields and stress distributions for bearing capacity and earth pressure problems due to the inhomogeneity of the medium will be presented. Finally a recommendation is made concerning the treatment of bearing capacity and retaining wall problems based on the construction of an average failure function from the site boring tests results.

**METHOD OF ANALYSIS**

An element of soil under limit equilibrium condition is considered in Fig. 1 in which the normal stresses are indicated by $\sigma_x$ and $\sigma_z$ and $\tau_{xz}$ denotes shear stress acting on the element. The soil mass is considered to behave as a rigid plastic medium for which a nonlinear failure criterion is assumed. This element of soil satisfies the equations for equilibrium

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0
\]

\[
\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma
\]

as well as the failure function

\[
\tau = \tau(x, z, \sigma)
\]

where $\gamma$ denotes the specific gravity of the soil mass and it depends on depth $z$.

The nonlinear variations of shear stress $\tau$ with normal stress $\sigma$ for any given point $(x, z)$ of the considered medium are shown on the Mohr Circle representation, Fig. 2, for the state of stress of that element. If $\theta$ denotes the angle between the direction of the principal axis and the $x$ axis, then from Fig. 2 the following expressions can be written

\[
\sigma_x = u - f \cos 2\theta - \phi
\]
\[ \sigma_i = u + f \cos 2\theta - \phi \]  
\[ \tau_{ss} = f \sin 2\theta. \]  

The terms \( u, f \) and \( \phi \) are of the form of stress and they are described in the figure. The two directions PB and PC are the line along which slip takes place. Thus if at any point \((x, z)\) in the region of limit equilibrium values of \(\theta\) and \(u\) are given, then slip directions as well as stresses \(\sigma, \sigma_i\) and \(\tau_{ss}\) can be determined.

**SLIP LINE FIELD**

Since shear stress \(\tau\) is a function of \(x, z\) and \(\sigma\) then from the definitions given in Fig. 2, it is clear that

\[ u = u(x, z, \sigma) \]  
\[ f = f(x, z, \sigma) \]  
\[ \phi = \phi(x, z, \sigma). \]

Elimination of \(\sigma\) from above equations will result in the following expressions for \(f\) and \(\phi\):

\[ f = f(x, z, u) \]  
\[ \phi = \phi(x, z, u) \]

which are in fact the known yield property functions for a given yield equation \(\tau = \tau(x, z, \sigma)\). The solution to the problem of limit equilibrium ends in giving:

\[ u = u(x, z) \]  
\[ \theta = \theta(x, z) \]

and consequently

\[ f = f(x, z) \]  
\[ \phi = \phi(x, z). \]

Now if operators \(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}, \frac{\partial}{\partial u}\) are defined as the partial derivatives of equations (10) and (11) and \(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\) and \(\frac{\partial}{\partial u}\) are the partial derivatives for equations (12)-(13), it is clear that:

\[ \frac{\partial f}{\partial x} = \frac{\delta f}{\delta x} + \frac{\delta f}{\delta u} \frac{\partial u}{\partial x} \]  
\[ \frac{\partial \phi}{\partial x} = \frac{\delta \phi}{\delta x} + \frac{\delta \phi}{\delta u} \frac{\partial u}{\partial x}. \]

Similar partial derivatives can be taken with respect to \(z\) or \(u\). It should be noted that \(\frac{\delta}{\delta x}, \frac{\delta}{\delta z}, \frac{\delta}{\delta u}\) are known yield property functions. Substitution of equations (4) through (6) in (1) and (2) will result in the following differential equations:
\[
\frac{\partial u}{\partial x} \left[ 1 - \frac{\partial \phi}{\partial u} + \cos 2\theta \frac{\partial f}{\partial u} \right] + \frac{\partial u}{\partial z} \left[ \sin 2\theta \frac{\partial f}{\partial u} \right] + \frac{\partial \theta}{\partial x} \left[ -2f \sin 2\theta \right] + \frac{\partial \theta}{\partial z} \left[ 2f \cos 2\theta \right] \\
= \frac{\partial f}{\partial x} - \cos 2\theta \frac{\partial f}{\partial x} - \sin 2\theta \frac{\partial f}{\partial z}
\]
(18)

\[
\frac{\partial u}{\partial x} \left[ \sin 2\theta \frac{\partial f}{\partial u} \right] + \frac{\partial u}{\partial z} \left[ 1 - \frac{\partial \phi}{\partial u} - \cos 2\theta \frac{\partial f}{\partial u} \right] + \frac{\partial \theta}{\partial x} \left[ 2f \cos 2\theta \right] + \frac{\partial \theta}{\partial z} \left[ 2f \sin 2\theta \right] \\
= \frac{\partial \phi}{\partial z} + \cos 2\theta \frac{\partial f}{\partial x} - \sin 2\theta \frac{\partial f}{\partial z} + \gamma
\]
(19)

Furthermore, the total differentials of u and \( \theta \) are:

\[
\frac{\partial u}{\partial x} \, dx + \frac{\partial u}{\partial z} \, dz = du
\]
(20)

\[
\frac{\partial \theta}{\partial x} \, dx + \frac{\partial \theta}{\partial z} \, dz = d\theta
\]
(21)

From the above four relations the characteristic equation for the four unknowns \( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial z}, \frac{\partial \theta}{\partial x}, \frac{\partial \theta}{\partial z} \) is obtained by equating the determinant of coefficients to zero, thus

\[
\left( \frac{dx}{dx} \right) \left[ \frac{\partial f}{\partial u} + \cos 2\theta \left( 1 - \frac{\partial \phi}{\partial u} \right) \right] - 2 \frac{\partial f}{\partial u} \sin 2\theta \left( 1 - \frac{\partial \phi}{\partial u} \right) + \left[ \frac{\partial f}{\partial u} - \cos 2\theta \left( 1 - \frac{\partial \phi}{\partial u} \right) \right] = 0
\]
(22)

and by letting

\[
\cos 2\mu = \frac{\partial f}{\partial u} \left( 1 - \frac{\partial \phi}{\partial u} \right)
\]
(23)

the two characteristic directions are deduced to be

\[
\frac{dz}{dx} = \tan(\theta + \mu)
\]
(24)

\[
\frac{dz}{dx} = \tan(\theta - \mu).
\]
(25)

It is clear from Fig. 2 that

\[
\tan \phi = \frac{\partial \tau}{\partial \sigma}
\]
(26)

\[
f = r \sqrt{1 + (\partial \tau / \partial \sigma)^2}
\]
(27)

\[
u - \phi = a + \frac{\partial \tau}{\partial \sigma}
\]
(28)

and
\[ \frac{\delta f}{\delta a} \frac{\delta \tau}{\delta \sigma} + \frac{\delta^2 \tau}{\delta \sigma^2} \]  \quad (29) \\
\frac{\delta}{\delta a} (\mu - \phi) = 1 + (\delta \tau/\delta a)^2 + \frac{\delta^2 \tau}{\delta a^2}. \quad (30)

Dividing equation (29) by (30) we obtain
\[ \frac{\delta f}{\delta u} (1 - \frac{\delta \phi}{\delta u}) = \frac{(\delta \tau/\delta a)}{\sqrt{1 + (\delta \tau/\delta a)^2}} \sin \phi. \quad (31) \]

Comparison of equation (31) and (23) would give us the following value for \( \mu \)
\[ \mu = \frac{\pi}{4} - \frac{\phi}{2}. \quad (32) \]

Thus it is concluded that the slip line directions are the same as the characteristic directions.

**ON THE NATURE OF DIFFERENTIAL EQUATIONS OF LIMIT EQUILIBRIUM ON CHARACTERISTIC LINES**

Multiply equation (18) by \( \sin (\theta \pm \mu) \) and equation (19) by \( -\cos (\theta \pm \mu) \) and then by addition and further simplification the following equations will result:
\[ \frac{\partial u}{\partial x} - \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta x} + \tan (\theta - \mu) \left[ \frac{\partial u}{\partial z} - \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta z} \right] \]
\[ = \cot 2\mu \frac{\delta f}{\delta x} \left[ \sin (\theta + \mu) \frac{\delta \phi}{\delta x} - \cos (\theta + \mu) \frac{\delta \phi}{\delta z} \right] \cos (\theta - \mu) \frac{\delta f}{\delta z} \equiv a \quad (33) \]
\[ \frac{\partial u}{\partial x} + \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta x} + \tan (\theta + \mu) \left[ \frac{\partial u}{\partial z} + \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta z} \right] \]
\[ = \cot 2\mu \frac{\delta f}{\delta x} \left[ -\sin (\theta + \mu) \frac{\delta \phi}{\delta x} + \cos (\theta - \mu) \frac{\delta \phi}{\delta z} \right] + \cos (\theta + \mu) \frac{\delta f}{\delta z} \equiv b. \quad (34) \]

On the characteristics \( \frac{dz}{dx} = \tan (\theta \pm \mu) \) equations (32) and (33) will take the following forms:
\[ \frac{\partial u}{\partial x} - \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta x} \]  \( dx + \left[ \frac{\partial u}{\partial z} - \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta z} \right] dz = adx \]
for \( \frac{dz}{dx} = \tan (\theta - \mu) \) \quad (35)
\[ \frac{\partial u}{\partial x} + \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta x} \]  \( dx + \left[ \frac{\partial u}{\partial z} + \frac{2f}{\delta f/\delta u} \cot 2\mu \frac{\delta \theta}{\delta z} \right] dz = bdx \]
for \( \frac{dz}{dx} = \tan (\theta + \mu) \) \quad (36)
Now if we compare the above equations with those given by (20) and (21), the following expressions would yield for $du$ and $d\theta$ on the two characteristic lines:

$$du - \frac{2f}{\delta f/\delta u} \cot 2\mu dt = adx \quad \text{on} \quad \frac{dz}{dx} = \tan(\theta - \mu) \tag{37}$$

$$du + \frac{2f}{\delta f/\delta u} \cot 2\mu d\theta = bdx \quad \text{on} \quad \frac{dz}{dx} = \tan(\theta + \mu) \tag{38}$$

which are in fact the variations of $u$ and $\theta$ on the characteristics. Now it is evident that, the method of characteristics is most suitable for obtaining $u$ and $\theta$.

In Fig. 3 points B and C are located on the characteristics lines $\frac{dz}{dx} = \tan(\theta - \mu)$ and $\frac{dz}{dx} = \tan(\theta + \mu)$ respectively and A is the point of intersection of these two characteristics.

If points A, B and C are taken close to each other, one can apply the method of finite element on these characteristic directions and equations (37) and (38) to obtain:

$$x_{i,j} = x_{i-1,j} + \frac{z_{i-1,j} - z_{i,j-1} + [x \tan(\theta - \mu)]_{i-1,j} - [x \tan(\theta + \mu)]_{i,j-1}}{\tan(\theta + \mu)_{i-1,j} - \tan(\theta - \mu)_{i,j-1}} \tag{39}$$

$$\theta_{i,j} = \theta_{i-1,j} + \frac{[\theta d]_{i-1,j} - [z_{i,j-1} - z_{i,j}]}{[x d]_{i-1,j} - [x z]_{i,j-1}} \tag{40}$$

$$\theta_{i-1,j} = \theta_{i-1,j-1} + \frac{[\theta d]_{i-1,j-1} - [z_{i,j} - z_{i,j-1}]}{[x d]_{i-1,j} - [x z]_{i,j-1}} \tag{41}$$

Subscripts $(i-1, j)$, $(i, j)$ and $(i, j-1)$ respectively refer to points B, A, and C, and the term $d$ is equal to $\frac{2f}{\delta f/\delta u} \cot 2\mu$.

As it is shown in equations (39) through (42), once the yield parameters $(x, z, \theta, u)$ are known on two distinct points such as B and C (not being on the same characteristic line) then the yield parameters can be evaluated for point A which is the intersection of two characteristics passing through B and C. Thus the solution of the limit equilibrium problem for different boundary value situations such as Cauchy, Goursat and Mixed boundary values is obtained as demonstrated below.

**BEARING CAPACITY PROBLEM**

The problem of bearing capacity of nonhomogeneous soil is studied by considering a normal surcharge $p$ acting on OA (Fig. 4) on which line the shear is zero. The resulting plastic equilibrium zone OAA'B'BO' is composed of three distinct regions, each constituting a specific boundary value problem:

*Region I* represents a Cauchy boundary value problem in which along the boundary OA, $z=0$, $\theta=0$ and also $x$ and $u$ are known. Hence, by using the recurrence formulas (39) through (42) the values $x$, $z$, $u$ and $\theta$ can be calculated for points on two sets of characteristics, one set designated by the angle $\theta - \mu$ and the other set by $\theta + \mu$. Calculations of the successive points in the region would end with the determination of the values of $x$, $z$, $u$, and $\theta$. The points on OA are designated by $x$. As a result of these calculations, the region OA is divided into two parts. The region to the right of OA is called a Cauchy region in which $\theta = 0$, and the region to the left of OA is called a Goursat region in which $\theta = \pi$. Thus the solutions of the limit equilibrium problems for different boundary value situations such as Cauchy, Goursat and Mixed boundary values are obtained as demonstrated below.
θ and u along the line OA' which is common to both regions I and II.

Region II is a Goursat type boundary value problem in which the values of x, z, θ and u are known on the common characteristic OA' and they are in fact the boundary values of the Goursat problem. On the small curve OO' around the singular point, x and z are zero and u is given by the expression

\[ u = u_0 e^{2\theta \tan \delta} \]

where θ varies from zero on OA' to π/2 on OB'. Again by employing the recurrence formulas the values of x, z, θ and u on the two families of characteristics in the OA'B'O region are obtained.

Region III constitutes a mixed boundary value problem where the values of x, z, θ and u are known on OB' as the result of our calculations in region II. Also, on O'B', z=0 and θ=π/2. Therefore, solutions are obtained for this region as a consequence of which the ultimate bearing capacity q along O'B is determined.

The numerical examples are carried out in this work by an 1130 computer using the procedure shown in Fig. 5. The distance OA is divided into M−1 divisions of equal dx, starting with the point (M, M) at 0 and going to the point (1, 1) at A. The small semi-loop OO' around the singular point is also divided into L−1 sectors of equal dθ, the first point being (M, M) again at O and the last one being designated by (M+L−1, M) at O'. Thus any point in the plastic equilibrium region is now specified by two characteristics I and J. The I characteristic makes an angle of θ+μ with the x axis and the J one forms
the angle $\theta - \mu$.

Now, by employing the recurrence formulas, using the known values of $x$, $z$, $\theta$, $u$ of points (M, M) and (M-1, M-1) those of the point (M, M-1) are calculated. The same process is continued up to point D, i.e. (M+L-1, M-1). Furthermore we know that at point E, $x=0$ and $\theta = \pi/2$, then by using the relations on the characteristic DE the necessary information for the location (M+L, M-1) is calculated. After all the locations on the characteristic M-1 are exhausted then calculations will proceed with those on M-2, M-3, …… and finally the entire plastic equilibrium region is covered.

**EARTH PRESSURE PROBLEM**

The problem of a vertical retaining wall with constant wall friction angle $\delta$ is treated in this work. However the method of calculations presented in this section is only for the passive case because that of active follows similarly.

Consider the surface OA of a vertical wall under the normal pressure $p$ as suggested in Fig. 7. Again, the zone OAA'B'B'O' is composed of three distinct regions where I and II are very similar to the corresponding regions discussed for the bearing capacity problem. Although region III represents again a mixed boundary value problem, it can not be treated in the same way as that of conventional type retaining walls. The obvious deviation is due to nonlinearity of the failure function as indicated in Fig. 6. It is evident from this figure that the apparent angle of internal friction $\phi$ and the apparent cohesion $c$ both vary with the stress $\sigma$.

\[ \text{Fig. 6. Variations of } \phi \text{ and } c \text{ with } \sigma \]

In region III (Fig. 7) because of the wall friction $\delta$ the values of $\theta$ along O'B' are obtained from

\[ \theta = \frac{1}{2} (\delta + \varphi) \]  \hspace{1cm} (43)

where $\varphi$ is given by

\[ \sin \varphi = \frac{\frac{u}{\varphi} \sin \delta}{f}. \]  \hspace{1cm} (44)

Above equations yield an implicit relation between $u$ and $\theta$ for the points on the wall at any depth $x$.

The computational calculations are carried out by dividing OA and O'O' into M-1 equal intervals and L-1 equal sectors respectively, see Fig. 7. The computation starts on the characteristic M-1 and proceeds to point D on the last sector providing the values of $x$, $z$, …
$u, \theta$ for that location. For the last location on the same characteristic (point E) the value of $z$ is calculated from the expression $dz = \tan(\theta - \mu) dx$. Then equation 37 together with the implicit relation between $u$ and $\theta$ would lead to a trial and error procedure in a subroutine to calculate the values of $\theta$ and $u$ at point E. The aforementioned procedure is extended to the previous characteristics $M = 2, M = 3, \ldots$ and consequently the entire plastic equilibrium region is covered.

RESULTS

The results of this study should prove to be of particular importance for the engineering design in nonhomogeneous soil media, as the subsequent examples would indicate a considerable amount of deviation in stress distributions as well as slip line fields from those predicted by the existing theories of homogeneous soils, references (5) and (8). In the following sections the numerical calculations of bearing capacity and retaining wall problems are carried out for a particular class of nonhomogeneous medium. However the method of formulation is quite general and it can be easily adopted to any type of nonhomogeneous soils encountered in practice.

As it was pointed out earlier, the soil mass used in this work is assumed to satisfy the failure function $r = r(x, z, \sigma)$, and as an example of this general failure criterion several numerical examples are presented for the particular type of soils obeying the law

$$r = z + (0.577 - 0.309z) \sigma - ze^{-1.664} \sigma$$  \hspace{1cm} (45)
where $\tau$, $\sigma$ and $z$ are in their nondimensional forms. This type of soil mass is of significant practical interest because at the surface, $z=0$, it behaves like sand having an angle of internal friction of 30 degrees. Whereas at the depth $z=1$ with large values of $\sigma$ it resembles a silty clay with an apparent cohesion of unity and 15 degrees apparent internal friction, as shown in figures (8) and (9).

![Fig. 8. Yield criterion at $z=0$](image1)

![Fig. 9. Yield criterion at $z=1$](image2)

The specific gravity of the soil mass is assumed to vary linearly with the nondimensional depth $z$ in accordance with the relation

$$\gamma = 1.6 + 0.04z.$$  \hspace{1cm} (46)

**Bearing Capacity:** The problem of bearing capacity (Fig. 4) was solved for several values of surcharge, i.e. $p=0.01$, 0.25, 0.5 and 0.75 kg/cm$^2$ and the consequent slip line fields as well as the bearing stress $q$ were determined. A typical slip line field is presented in Fig. 10 which is for the case of $p=0.01$ kg/cm$^2$. Also, the values of ultimate bearing stress $q_{\text{max}}$ versus the bearing dimension $l$ are presented in Fig. 11 for the aforementioned values of surcharge. It is clear from these figures that the ultimate bearing capacity $q_{\text{max}}$ is not proportional to $l$ as the proportionality is assumed for the case of homogeneous soils by Biarez (2) and others. Furthermore, it is demonstrated that the simple bearing capacity formula (Ref. (5))

$$q_{\text{max}} = cN_v + pN_v + \gamma N,$$  \hspace{1cm} (47)
is no longer applicable to the bearing stress evaluations when the medium is not homogeneous.

From the bearing capacity curves corresponding to $\rho = 0.50$, $0.75 \text{ kg/cm}^2$ surcharge, it is observed that at larger values of $l$ and $\sigma$ the cohesive component of bearing capacity increases with $l$ whereas the frictional component decreases. These behaviors are due to the physical properties of the medium for which the apparent cohesion $c$ is increased and the apparent angle of internal friction $\phi$ is decreased with $l$. However at low values of stress $\sigma$ the cohesive component is reduced and the frictional component is increased, the effects of which are demonstrated by the continuous rise in the $0.01 \text{ kg/cm}^2$ surcharge curve of Fig. 11. Thus it can be concluded that the estimates of the bearing capacity of a nonhomogeneous soil based on the use of the average values of $c$ and $\phi$ in equation (47) can be grossly in error.

To demonstrate this further, the ultimate bearing capacity $q_{\text{max}}$ is calculated from equation (47) for which the values of $N_c$, $N_\phi$, and $N_\gamma$, based on the mid-depth values of $\phi$, $c$ and $\gamma$,
are taken from Table 6-6 presented in reference (5). The variations of $(q_{\text{max}})_{\text{conventional}}$ is plotted versus length for two values of surcharge $p$. These plots are then superimposed on those obtained by the theory presented in this work, Fig. 11. The comparison of these two sets of curves for $q_{\text{max}}$ indicates the significant difference and hence the inaccuracy of the conventional formula for use in evaluation of bearing capacity of nonhomogeneous soils with nonlinear failure criterion.

**Earth Pressure:** The problem of earth pressure was solved for a vertical wall with various values of wall friction $\delta = 1, 5, 10$ and $15$ degrees on which a uniform surcharge $p$ was applied, values of $p$ being from 0.01 kg/cm² to 1 kg/cm². For all the cases of $p$ and $\delta$ both active and passive regions were studied. A typical slip line field for the passive case is shown in Fig. 12 which is drawn for $\delta = 10^\circ$ and $p = 0.01$ kg/cm². Fig. 13 represents the variation of the normal wall pressure versus depth for different values of surcharge at $\delta = 15^\circ$, whereas Fig. 14 shows that for different values of friction $\delta$ at 0.01 kg/cm² surcharge.

Similar results are presented in Figs. 15 through 17 for the active case. For this case the direction of $\delta$ is opposite to that of passive shown in Fig. 7.

The effect of inhomogeneity of soil medium is clearly pronounced both in the slip line fields and the resulting stresses. The slip lines are no longer straight even in the region which corresponds to Cauchy boundary value problem. The angle of slip lines with horizontal approaches the value of $60^\circ$ at the surface and it is increased at lower depth. As it is shown the slip line curvatures are upward in zone I of passive region and this is of course in conformity with the soil property predictions. Furthermore the disturbed depth is smaller for the passive case than that of active.

The pressure distribution graphs, Figs. (13) and (16), show that at larger depth the passive pressure is increased and the active pressure is decreased. These are, of course, due to physical properties of soil mass which exhibits more cohesion at larger values of $z$. Thus, the load influence of surcharge goes as far as a limited depth in the active case and that influence becomes less pronounced at deeper depths. This form of normal stress distribution as presented in Figs (16) and (17) appears in braced trench excavation the reason for which being different displacement for the bracing and different developed $\phi$ for the soil.
Fig. 13. Earth pressure: plots of $\sigma_x - z$ for passive case of different values of surcharge at $\delta = 15^\circ$.

Fig. 14. Plots of $\sigma_x - z$ for passive case for different values of wall friction angle at $p = 0.01$.

Again to demonstrate the inapplicability of the conventional method for evaluating the normal pressure distribution of the retaining wall problems in nonhomogeneous soils the middepth properties of the soil are supposed to be representative for the entire mass, i.e. the values of the apparent cohesion $c = 0.5 \text{ kg/cm}^2$ and the apparent angle of internal friction $\phi = 22.5^\circ$ are assumed to represent the soil for the case of $\delta = 0$. Then the normal pressure distribution for the passive case is calculated from the following expression (see Ref. (5))

$$\sigma_x = \sigma_z \tan^2 \left( 45 + \frac{\phi}{2} \right) + 2c \tan \left( 45 + \frac{\phi}{2} \right).$$

The conventional results are then superimposed on Fig. 14 in which the gross differences are clearly demonstrated. Curves for assuming the upper layer and lower layer to represent the soil are also plotted on the same figure.

CONCLUSIONS

The following conclusions are derived from the analytical study presented in this work concerning the effect of inhomogeneity of soil on bearing capacity and earth pressure problems.

1. The limit equilibrium of nonhomogeneous soil medium satisfying a nonlinear yield criterion $\tau = \tau(\sigma, x, z)$ is proved to admit itself to a solution by the method of characteristics. Furthermore the characteristic directions and the slip line directions are shown to coincide.
2. The limit equilibrium analysis of nonhomogeneous soil medium are shown to be extendable to the solutions of bearing capacity and earth pressure problems in that medium.

3. For the particular nonhomogeneous soil mass assumed in this analysis it is clearly demonstrated that the slip line fields as well as the stress distributions can not be accurately determined from the conventional methods even if the analysis is based on the average values of $c$, $\phi$ and $\gamma$.

4. When site borings indicate inhomogeneity either due to changes in layers or different compactive efforts, it is recommended that an average failure criterion be constructed from the site tests and then by a similar analysis the stress distribution be obtained.
ACKNOWLEDGEMENTS

The authors acknowledge with gratitude the kind service of Pahlavi University Computer Center in connection with the computational part of this work. The financial support of Pahlavi University Research Council and the kind assistance of Miss Rohan Kohanim are also acknowledged.

NOTATION

The following symbols have been used in this work

- $a$ = a known function
- $b$ = a known function
- $c$ = cohesion
- $f$ = stress (see Fig. 2)
- $f'$ = location on slip line field
- $f''$ = location on slip line field
- $L$ = location on slip line field
- $l$ = distance
- $M$ = location on slip line field
- $N_x$ = constant (see equation 47)
- $N_y$ = constant (see equation 47)
- $N_z$ = constant (see equation 47)
- $p$ = surcharge
- $q$ = bearing stress
- $u$ = stress (see Fig. 2)
- $x$ = horizontal coordinate
- $z$ = depth
- $\gamma$ = specific gravity of soil
- $\theta$ = angle between direction of principal axis
- $a$ = normal stress
- $\phi$ = stress (see Fig. 2)
- $\phi'$ = angle of internal friction
- $\mu = \frac{\pi}{4} - \phi$
- $\tau = \text{shear stress}$

REFERENCES


(Received January 11, 1978)