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PREDICTION OF TIP RESISTANCE FROM PILE DYNAMICS*

M. A. SATTER AND A. GHAHRAMANI

Department of Mechanical Engineering, and Department of Civil Engineering,
Shiraz University, Shiraz, Iran.

Abstract - This report undertakes to evaluate conditions at which input power to a pile is said to reach optimum level, and to study the dynamics of the pile if the input power level is increased beyond the optimum value. The study is confined to low frequency vibropile driving using closed tip model piles and fine grain sand. For the post optimum condition, the pile induces relatively large impactive reaction (tip resistance) from the soil, which may cause damage to the embedded tip. A simple nonlinear mathematical theory for the impactive reaction or tip resistance is proposed. Experimental and theoretical results have good agreement.

1. INTRODUCTION

Vibratory excitations, including impacts, are widely used in pile driving at the present time. Pure vibratory pile driving may be classified into low frequency and high frequency methods, depending on the frequency of the input force. Low frequency pile driving employs frequencies in the range 5-60 Hz which is substantially below the fundamental longitudinal frequency of a pile under consideration; the pile behaves as a rigid body. High frequency pile driving utilizes the resonant property of the pile which is driven and the frequency of excitation is a multiple of the pile fundamental longitudinal natural frequency. Of course, impactive pile driving embodies both low and high frequency methods since this form of excitation contains all the frequencies from zero to infinity.

Many investigators [1-2, 4-6, 8-10] have studied the dynamic behavior of pile and soil during vibropile driving in both low and high frequency ranges. The general conclusion of many of the studies is that pile-soil interaction is complicated due to both inhomogeneous characteristics of the soil and change of soil properties during vibrocompaction.

A computer program, CAPWAP, based on pile wave equation and measured pile top acceleration as the boundary condition of the pile motion has been developed [11, 12]. It

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assumes soil parameters to model soil resistance forces on the pile. The computed pile top force on a time base is compared with field measured pile force. The procedure is repeated adjusting the soil parameters until a good match is obtained. Recent studies include comparison of axial response for both impact and vibrodriven piles [13] and bearing capacity prediction of model piles under controlled conditions in the laboratory [14].

This report undertakes to study both experimentally and theoretically the low frequency vibropile driving problem. In this, the fastest rate and maximum depth of penetration achieved by a pile under a constant static surcharge depends on an optimum level of the input power. If the input power is increased beyond the optimum level, the pile does not penetrate the soil further, but undergoes steady state vibration. It is at this steady state condition that the pile exhibits relatively large impactive reaction, which is defined as the dynamic tip resistance. The side resistance appears to be negligible, probably because of the existence of minute clearance between the pile wall and the soil. Specifically, the experimental phase which precedes the theory involves the determination of (i) pile penetration depth for various surcharges, (ii) the minimum and optimum input force levels and (iii) dynamic tip resistance. The theoretical phase of study emphasizes the optimum and post optimum behavior of the pile. A nonlinear mathematical theory is proposed for the dynamic tip resistance. It utilizes the pile parameters and the experimental value of pile acceleration but not soil parameters directly. However, an important dynamic soil parameter may be predicted from the theory.

A somewhat parallel study of soil resistance in the case of impact driven piles has been made by Rausche, *et al.* [8]. The study uses two identical piles, one of which is free and the other embedded. A force on top of the free pile is measured from the velocity obtained through numerical integration of the acceleration records and this force is taken as the reference. In the mathematical model of the embedded pile, however, the soil resistance properties are continually adjusted *until the computed force at the pile top matches the* reference force mentioned earlier. The difference between the computed force and the actually measured force on top of the embedded pile is the force due to the soil resistance. This is also termed as *delta curve* which corresponds to the dynamic tip resistance curve in our present study on low frequency vibropile driving. The approach put forward in this paper is independent and unique.

2. EXPERIMENTS AND RESULTS

Laboratory tests have been conducted using a model pile and the brown subangular sand of Shiraz. The sand (soil) would correspond roughly to 16/40 U.S. standard sieve size. The experimental set-up is shown schematically in Fig. 1. An electrodynamic vibrator (shaker), supported by a flexible steel rope and balanced by a suitable counter-weight, drives the pile.

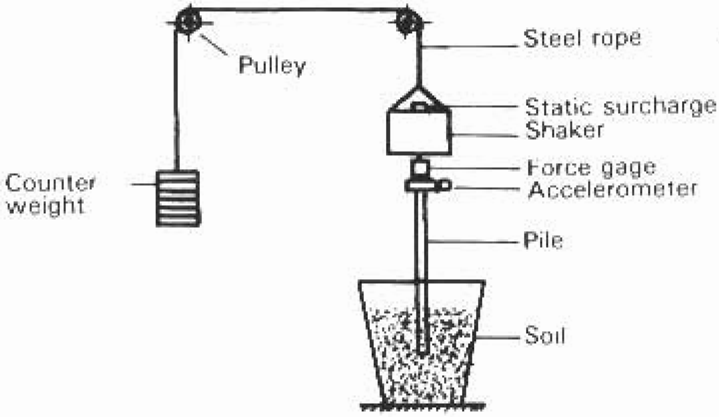

Fig. 1. Schematic diagram of the experimental set-up.

The steel rope is supported by two equal pulleys which are, in turn, mounted on ball bearings. This arrangement ensures the frictional force from the pulley to the rope to be negligible. The static surcharge is mounted on the shaker frame for convenience. A connecting device between the shaker and the pile was so designed as to minimize any eccentricity between the shaker head and the pile. The connecting device has the provision for mounting a force gage and an accelerometer for measurement purposes. The model pile has the following specification: material-mild steel, length-0.745 m, outer diameter-0.019 m, and inner diameter-0.0168 m. The fundamental longitudinal frequency of the pile is 1058 Hz. Both conically shaped with 60° included angle and flat end closed piles were used in the experiments. The equivalent mass of the pile is 0.711 kg.

The first set of experiments was conducted at an arbitrarily chosen frequency, 40 Hz, to observe the depth of penetration as a function of input power level in the shaker and the static surcharge.

Three different surcharges, namely, 8.9, 17.8, and 26.7 N have been used. The records are shown in Fig. 2. It is clear that the ultimate depth of penetration is dependent on both the input power level and the magnitude of static surcharge. Generally, there is greater penetration of the pile with heavier static surcharge. There is, however, a limit to the magnitude of static surcharge that may be applied to a vibrating pile; the limit is imposed by the shaker output force which must be greater than the static surcharge. There is also an optimum static surcharge which ensures deepest penetration of the pile for a certain level of input power. This fact can be noted from Fig. 2 where, for an arbitrary power level 3, the pile penetration reads 0.002, 0.03 and 0.05 m for applied static surcharges 8.9, 17.8, and 26.7 N, respectively. It is also observed from the curves in Fig. 2, that, for a certain static surcharge, there is an optimum power level for which the pile achieves certain depth

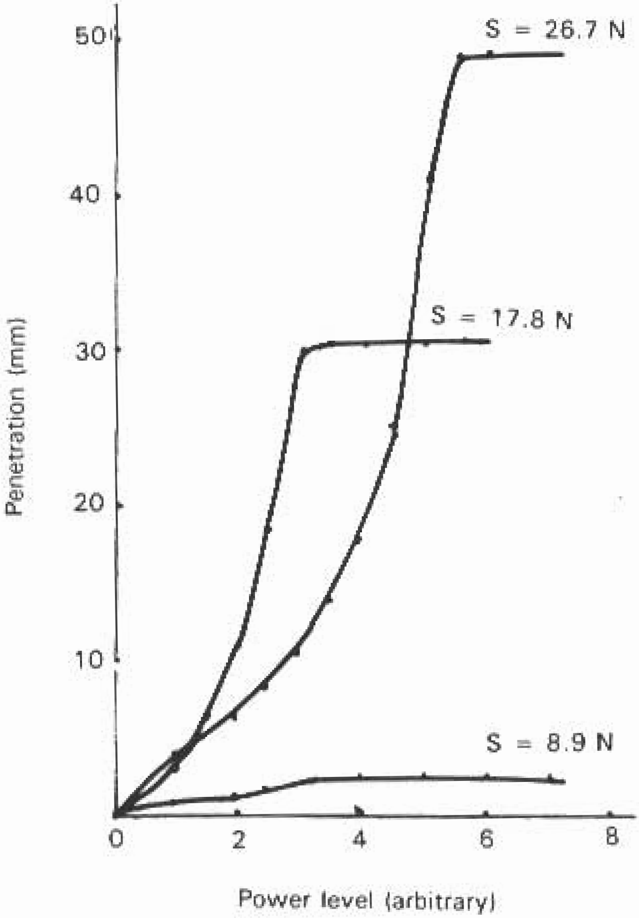

Fig. 2. Depth of penetration against static surcharge(s).

of penetration; any increase in the power level does not affect the pile penetration further. The second set of experiments was aimed at determining the level of minimum and optimum level of vector forces of the shaker necessary for vibro-pile driving. The minimum force level is defined as the force which causes the pile, under certain static surcharge, to simply vibrate and begin penetration very slowly into the soil. Optimum force level, on the other hand, is defined as the applied force under which the pile achieves ultimate penetration within the shortest possible time. It should be noted that, if the force level is increased beyond the optimum, no appreciable increase in the depth of penetration of the pile will result. Fig. 3 shows both the minimum and optimum level of the shaker force against various surcharges on the pile. The tests were conducted at 40 Hz which appeared to be a good choice for the pile-soil set-up mentioned earlier. It is clear and as expected, the minimum force level equals the static surcharge. However, the optimum force level

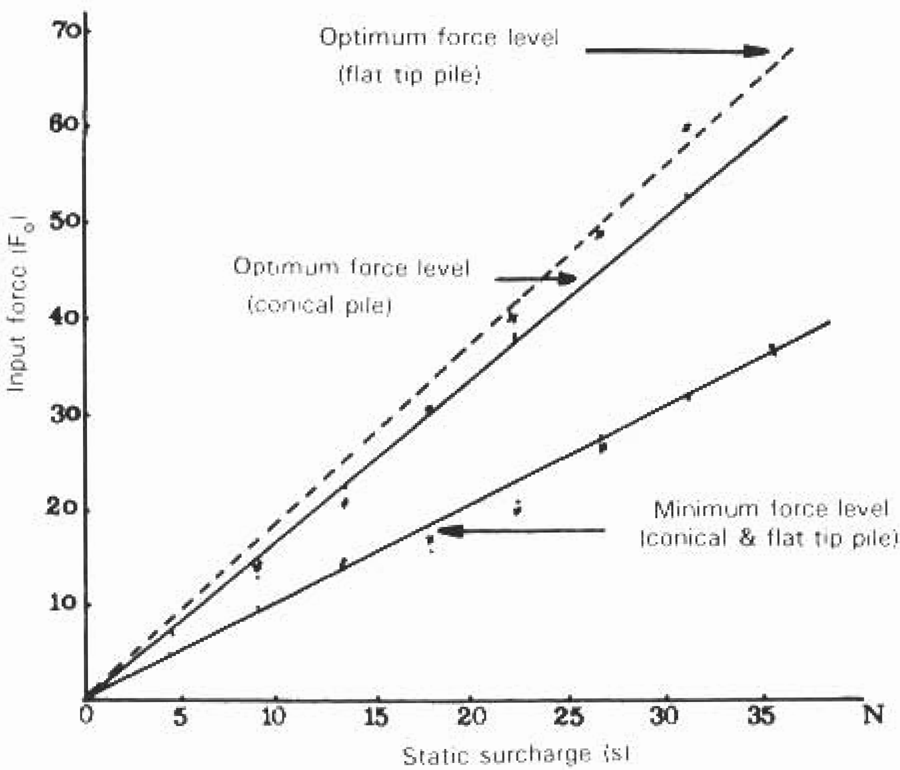

Fig. 3. Minimum and optimum force levels for various surcharges.

increases linearly with the static surcharge and the constant of proportionality for the present test arrangement is given by $\alpha = F_0/S \simeq 1.7$, where F_0 and S are the applied force and the surcharge respectively. Flat-tip pile requires slightly higher optimum force level. In this case $\alpha \simeq 1.9$.

The third phase of the experiments was related to the study of pile tip resistance after the pile achieves the ultimate penetration. It was mentioned earlier that the pile, under certain static surcharge, penetrates the soil to an ultimate depth, but no further penetration takes place even if the input power is increased substantially. The pile undergoes steady state vibration and the tip resistance at this condition is investigated here. The skin resistance is found to be negligible in the present investigation.

The tip resistance [4, 8, 10] of a pile may be expressed by $F(t) - m\ddot{x} = TR$, where TR = tip resistance, F(t) = input force, m = effective mass of the pile and its attachment, and $\ddot{x}$ = instantaneous acceleration of the pile. The above equation neglects skin friction or any other force on the pile. While the pile is outside the soil, there is no tip resistance, i.e., TR = 0, and the above equation takes the form $F(t) - m\ddot{x} = 0$. Examination of these two equations indicates that a force gage and an accelerometer are the only transducers necessary for measuring tip resistance experimentally.

In the present set-up, in addition to a force gage and an accelerometer, two charge amplifiers and a dual channel oscilloscope, with the facility for adding or subtracting the two signals, (F(t) and $m\ddot{x}$), were used. Care was taken to minimize phase distortion of the signals. With the pile vibrating outside the soil (TR = 0), the gains of the charge amplifiers were so adjusted as to produce two equal signals and these were combined to obtain null resultant on the oscilloscope. The resultant remains zero for any power input to the pile. As the pile begins to penetrate the soil, the resultant signal gives the tip resistance. The experiments were repeated for different surcharges and input signal frequencies. Piles with both flat and conically shaped ends have been used. In addition to recording the resultant, the two individual signals were also recorded while the pile vibrated in embedded and unembedded conditions. The signals were photographed for plotting on an enlarged scale.

The experimental results of steady state vibration of the pile under various configurations are plotted in Figs. 4-7. In all cases, the phase difference between signals of the applied force (curve A′) and the inertia force (curve B′) of the pile while vibrating outside the soil is 180° and their algebraic sum is zero. However, on embedment the inertia signal (B) changes its phase by almost 180° due to soil reaction on the pile and, when the two signals (A, B) are added the resulting signal (A + B) shows a series of peaks spaced by the period of excitation. Both the magnitude and the duration of the peaks indicate the pile reaction to be that with a hard spring. It is also clear from the curves (A + B) that the pile-soil interaction time is approximately one quarter of the period of excitation.

An important observation can be made from Fig. 4 where the presence of kinks is evident on the resultant curve (A + B).

The kinks indicate that the pile has not reached the steady state condition and that the rigid body penetration of the pile into the soil is still taking place. At this condition, the input power to the pile may be increased to achieve the ultimate penetration corresponding to the applied surcharge.

Figs. 4 and 5 show the results for different static surcharges applied to the conically shaped pile. Pile tip resistance appears to increase proportionately with the static surcharge. However, it should be noted that for larger static surcharge greater power input is also required. Therefore, it is probably not the static surcharge but greater power input which increases the tip resistance. A comparison between Figs 4 and 6 makes this more evident. For a surcharge, 8.9 N, and applied vector force, 24.45 N, (Fig. 4) the tip resistance is higher than that for the pile with surcharge 8.9 N and applied vector force 13.93 N (Fig. 6). Also, from Fig 5 in which the surcharges differ but input forces are of the same order, the tip resistances have similar magnitudes.

The corresponding results for the flat end pile are shown in Figs. 6-7. Generally, greater input of power is required to achieve the steady state condition. The tip resistances are also higher than those for the corresponding conical end pile. Probably, the conically shaped pile produces a better cushioning effect than a flat tip pile.

The effect of input frequency in the range 30-60 Hz on the pile tip resistances (Figs. 4 and 5) is not significantly apparent.

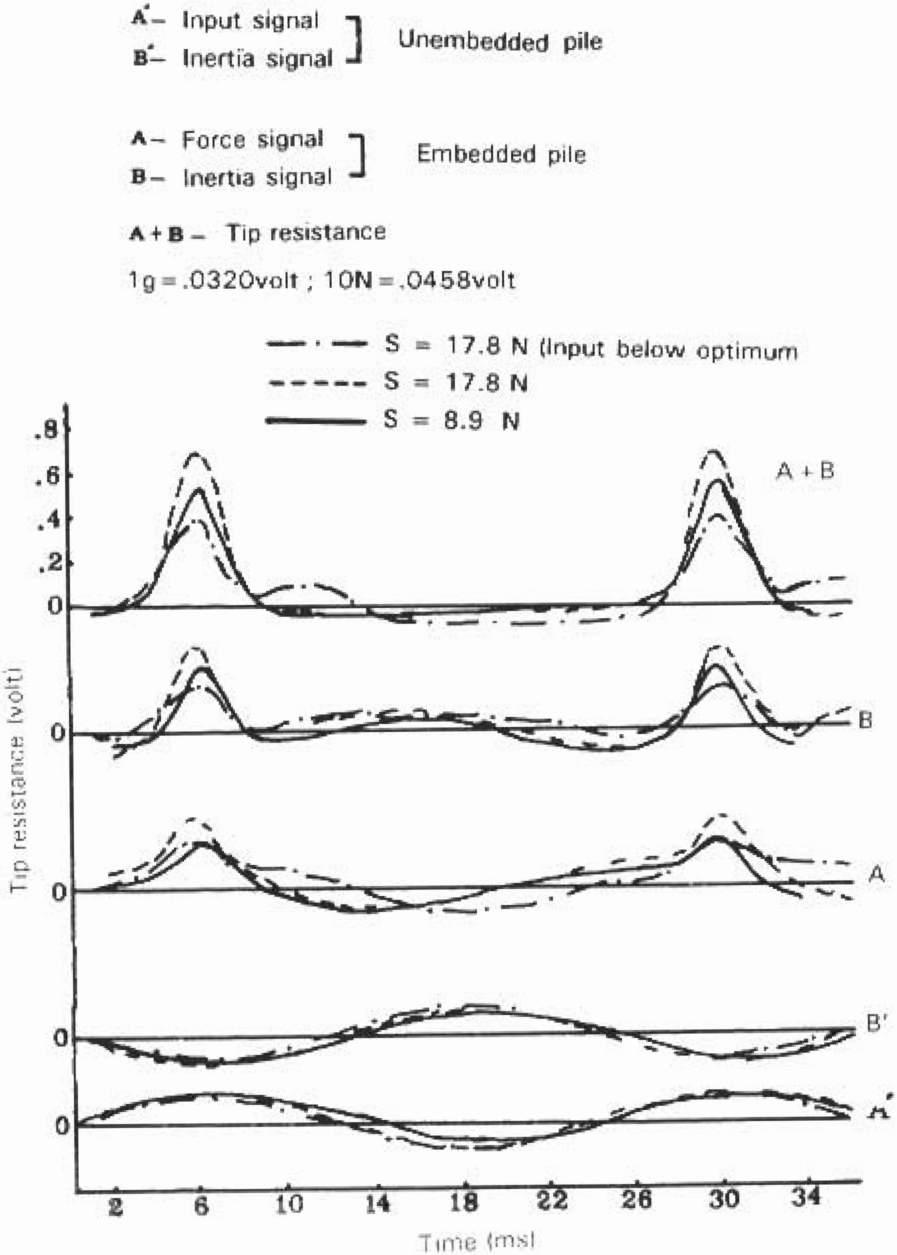

Fig. 4. Tip resistance of conical pile; F = 40 HZ.

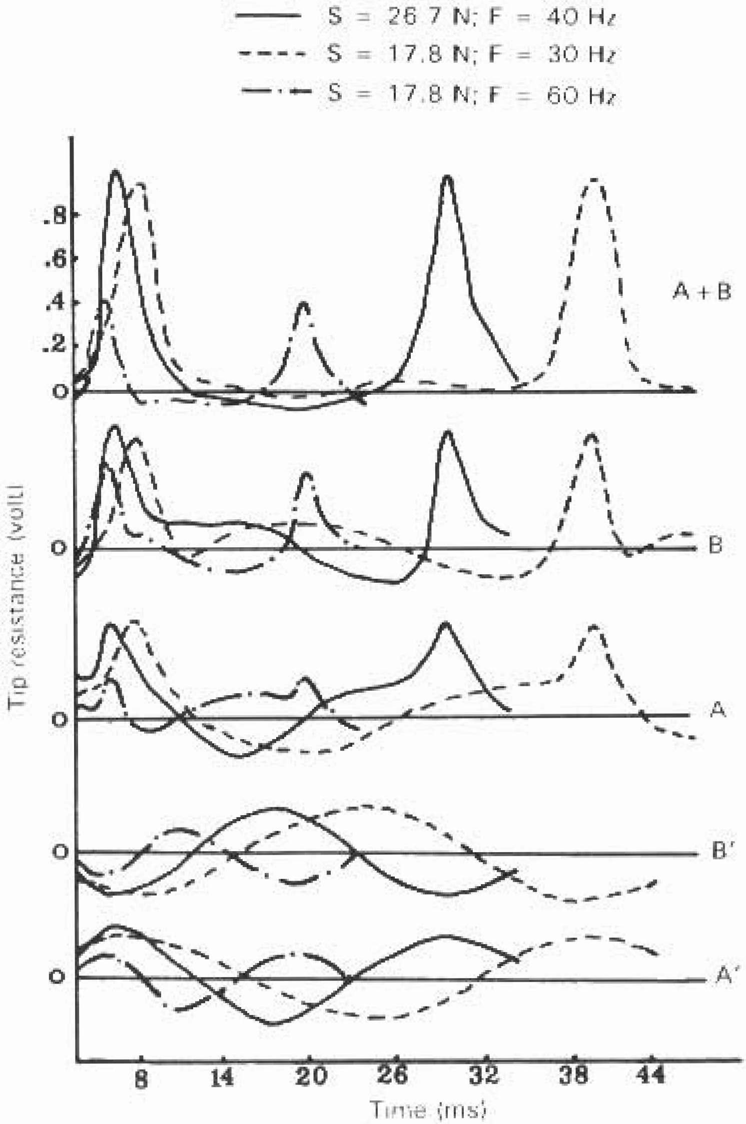

Fig. 5. Tip resistance of conical pile.

3. THEORY

It has been mentioned earlier that the model pile undergoes steady state vibration after it achieves the ultimate penetration which depends on the magnitude of the static surcharge and a certain power input to the pile. At this state, if the input power is increased, the vibrational amplitude increases but the pile does not penetrate the soil further. The pile experiences impactive resistance from the soil and the duration of pile-soil interaction is relatively small compared to the period of input excitation.

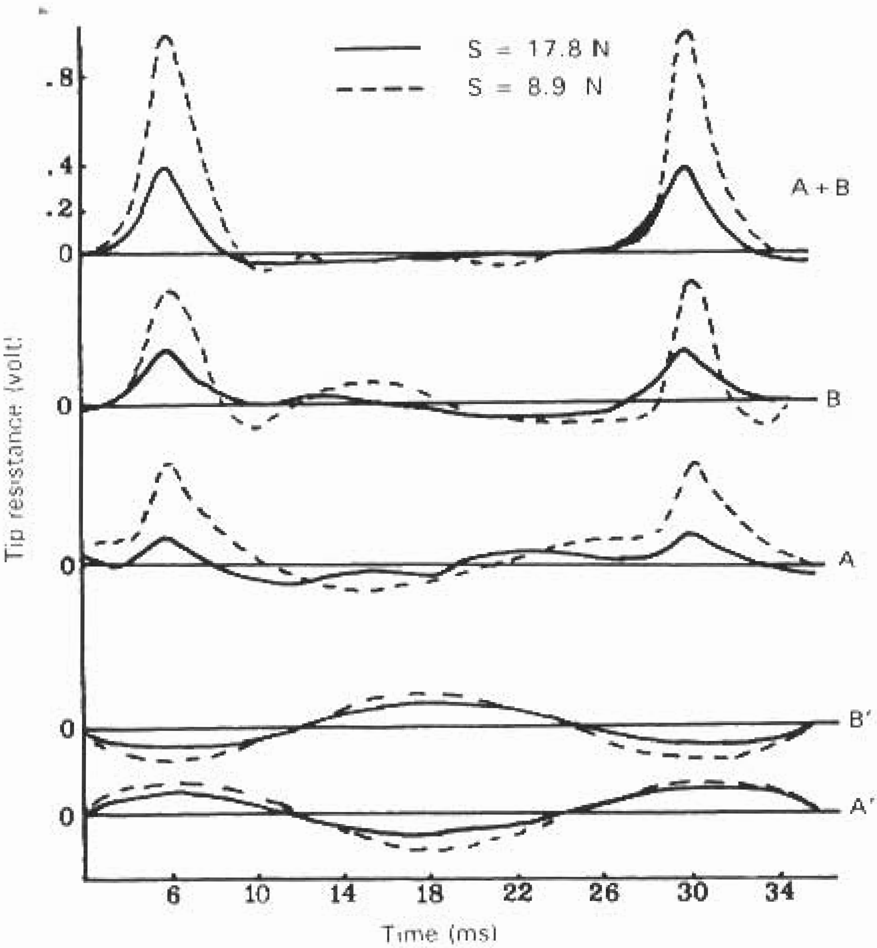

Fig. 6. Tip resistance of flat end pile, F = 40 HZ.

Fig. 8 shows a pile-soil interaction model. The skin resistance, which is small in the present investigation, is neglected. It has been found from random checks of the experimental data that the maximum tip resistance, TR, is proportional to the power three of the maximum displacement.
It is thus reasonable to assume

$$TR = Rx^3 \tag{1}$$

where R is a dynamic soil parameter to be determined. It is also assumed that the pile-soil interaction takes place only during the period the pile tip is below the mean vibrational level. Thus, R can have certain magnitude (R > 0) during the pile-soil interaction, otherwise R will be assumed to be zero. In view of this, Eq. (1) should be modified by introducing a filter function [15], i.e.,

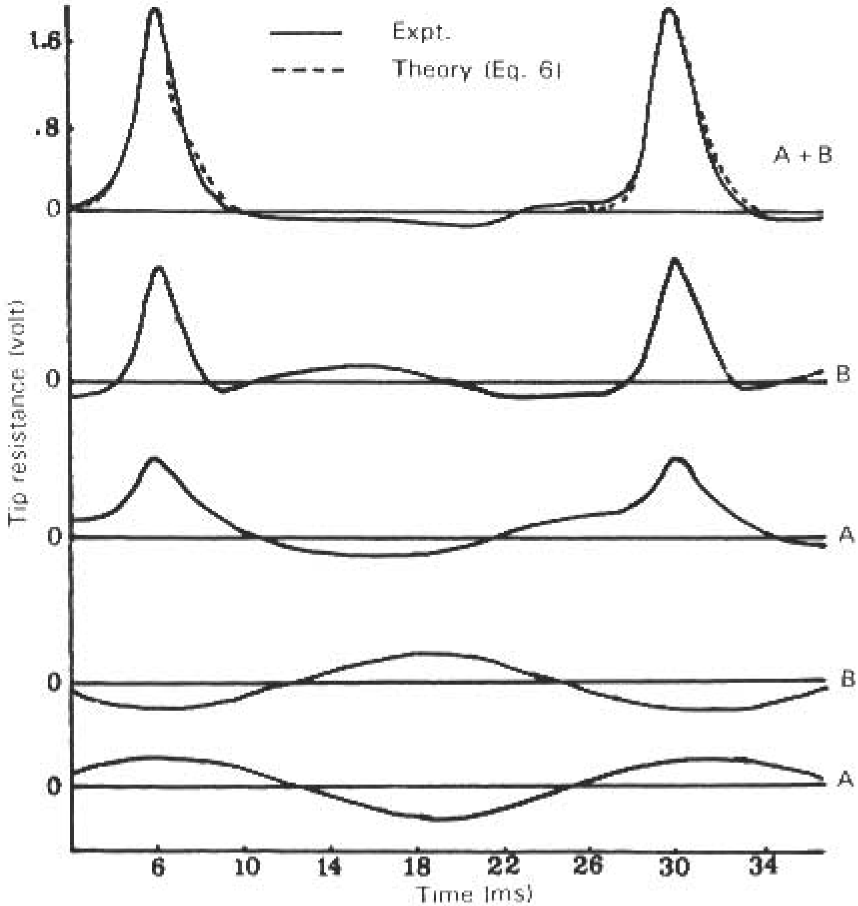

Fig. 7. Tip resistacne of flat end pile, S = 26.7N, F = 40 HZ.

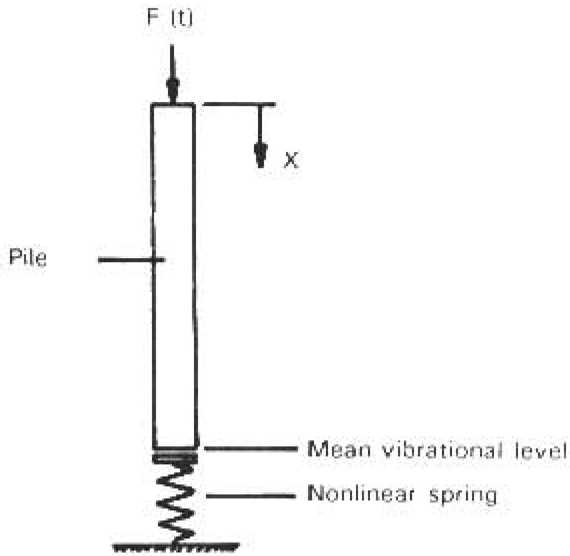

Fig. 8. Theoretical model of pile vibration.

$$TR = R[H(t-t_1) - H(t-t_2)]x^3 \tag{2}$$

where H(t - t′) is a unit step function.

Neglecting the skin resistance, the steady state vibration of the model pile may be expressed as

$$m\ddot{x} + R[H(t-t_1) - H(t-t_2)]x^3 = S + F_o \sin \omega t \tag{3}$$

where m = mass of the pile, S = static surcharge, F_o = amplitude of shaker force and ω = excitation frequency.

Eq. (3) is nonlinear and a first order approximate solution may be obtained [3, 7] in the following way. Rewriting Eq. (3), adding $\omega^2 x$ to both sides of the equation and substituting $x(t) = a \sin \omega t$ into the right hand, we obtain

$$\ddot{x} + \omega^2 x = \frac{S}{m} + \left\{\frac{F_o}{m} + \omega^2 a - \frac{3Ra^3}{4m}[H(t-t_1) - H(t-t_2)]\right\} \sin \omega t$$
$$+ \frac{Ra^3}{4m}[H(t-t_1) - H(t-t_2)] \sin 3\omega t \tag{4}$$

where the amplitude, a, is unknown. To avoid a secular term we impose the condition

$$\frac{F_o}{m} + \omega^2 a - \frac{3Ra^3}{4m}[H(t-t_1) - H(t-t_2)] = 0 \tag{5}$$

Eq. (5) is significant in the sense that it relates two important unknown quantities, a and R. Normally, a can be measured and hence R can be expressed through other measurable quantities. Thus,

$$R = (4F_o + 4m\,\omega^2 a)/3a^3$$

or

$$Ra^3 = (4F_o + 4m\,\omega^2 a)/3 = TR_{max} \tag{6}$$

Returning to Eq. (4) which, after deleting the secular term, reduces to

$$\ddot{x} + \omega^2 x = \frac{S}{m} + \frac{Ra^3}{4m}[H(t-t_1) - H(t-t_2)] \sin 3\omega t \tag{7}$$

The general solution of Eq. (7) is given by

$$x(t) = C_1 \sin \omega t + C_2 \cos \omega t + Y \tag{8}$$

where C_1 and C_2 are constants which depend on certain conditions, and Y is a particular solution of Eq. (7). Henceforth, the filter function is omitted for simplicity, but it should be understood to be associated with R. Thus,

$$Y = \frac{S}{m\omega^2} - \frac{Ra^3}{32m\omega^2} \sin 3\omega t \tag{9}$$

The conditions for evaluating C_1 and C_2 are

$$at \quad t = T/4, \quad x(t) = a \qquad \dot{x}(t) = 0$$

where, T = periodic time. Then

$$C_1 = a - \frac{S}{m\omega^2} - \frac{Ra^3}{32m\omega^2} \tag{10}$$

and

$$C_2 = 0$$

Substituting Eqs. (9) and (10) into Eq. (8), we obtain

$$x(t) = \frac{S}{m\omega^2} + (a - \frac{S}{m\omega^2}) \sin \omega t - \frac{Ra^3}{32m\omega^2} (\sin \omega t + \sin 3\omega t) \tag{11}$$

The acceleration is given by

$$\ddot{x}(t) = -\frac{S}{m} - \omega^2 \left[(a - \frac{s}{m\omega^2}) \sin \omega t - \frac{Ra^3}{32m\omega^2} (\sin \omega t + 9 \sin 3\omega t) \right] \tag{12}$$

In the above equation the constant acceleration term is added to account for the static surcharge.

It is noted from the curves in Figs. 4-7 that the inertia force signal changes its phase by about 180° during the pile-soil reaction. Therefore, in computing the tip resistance using Eq. (12) the following restrictions must be observed,

$$\begin{aligned} TR &= F(t) - m\ddot{x}, \quad for \quad t_1 \le t \le t_2 \\ TR &= 0 = F(t) + m\ddot{x}, \quad for \quad t < t_1 \quad or \quad t > t_2 \end{aligned} \tag{13}$$

In the case when TR = 0, the amplitude a, must be calculated from the relation, $a = F_0/m\omega^2$.

The maximum tip resistance may easily be calculated from Eq. (6).

4. APPLICATION OF THE THEORY TO FIELD PROBLEMS

It was mentioned earlier that an analogous problem has been studied in reference [8], where the pile is impact driven, while in the present investigation the pile is driven by a low frequency harmonic force. The fact that the theory developed for the impactive soil reaction to the pile is also applicable to field problems can be demonstrated by computing the maximum value of the *delta curve* of Fig. 1 in reference [8]. Extensive comparison of the theory with results in the above-mentioned reference is not easy because of the lack of field pile acceleration records. From Eq. (6), $TR_{max} = 4/3\ (F_o + m\omega^2\ a)$. In Fig. 1 of reference [8], taken from curve b, $F_o = 511.75$ kN and from curve C, $4\ m\omega^2\ a/3 = 1000$ kN, then $TR_{max} = 1682$ kN. The maximum value from the *delta curve* is 1669 kN. The discrepancy between these two results is only 0.8%.

5. COMPARISON OF THEORY AND EXPERIMENT

Both theoretical and experimental results for various pile configurations are summarized in Table 1. Also, a theoretical sample curve (dashed), plotted using Eq. (6), is shown in Fig. 7. It can be seen that the theoretical and experimental values for the pile tip resistance have very close agreement. In cases where the input force level is equal or greater than the corresponding optimum level the discrepancy between theory and experiment is less than one per cent. As expected, greater discrepancy is apparent in those cases (Figs. 5-6) in which the input force level is less than the optimum value. Theory also confirms the experimental finding that the dynamic tip resistance does not depend directly on the static surcharge. This is evident from both Eqs. (6) and (13). The theory has been applied to a field problem taken from reference [8]. It is shown that the value of the *delta curve* predicted by the theory is within one per cent of the value reported in the said reference.

6. CONCLUSIONS

The studies on low frequency vibropile driving have provided several useful observations. The minimum vector force the shaker must supply before the pile begins penetration into the soil equals the static surcharge on the pile. Corresponding to each surcharge, there is also an optimum input force level which ensures quickest penetration of the pile into the soil. Optimum force is linearly proportional to the static surcharge. Generally, the optimum force level is greater for a flat tip pile than that of a coned tip pile.

The optimum force level may be assumed to be reached by the disappearance of *kink* on the tip resistance curve. If the input force level exceeds the optimum, the pile does not achieve greater depth of penetration, but undergoes steady state vibration. The soil reaction

Table 1. Comparison of Experimental and Theoretical Data on Pile Behavior.

Fig. No.	Type of Pile	Static Surcharge S (Newton)	Frequency F (Hz)	Force F_o (Newton)	Acceleration (g)	Expt. TR max (Newton)	TR max (Newton)	Discrepancy %	Duration of Interaction (ms)
4	Conical end	8.9	40	24.45	9.56	121.5	121.0	0.4	6.2
4	Conical end	17.8	40	28.93	12.66	157.17	156.46	0.4	6
4	Conical end	17.8	40	22.16	6.56	91.67	90.69	1	10
5	Conical end	26.7	40	47.84	18.33	235.76	234.52	0.5	6.4
5	Conical end	17.8	30	44.56	17.16	221.0	219.21	0.8	7.4
5	Conical end	17.8	60	25.19	13.94	144.1	163.32	13	4.2
6	Flat end	8.9	40	13.93	8.75	91.67	100.0	9	6
6	Flat end	17.8	40	31.76	17.81	216.14	208.84	3.4	5.6
7	Flat end	26.7	40	58.86	37.22	426.0	425.15	0.2	5.6

or the dynamic tip resistance is proportional to the cube of the pile steady state displacement and it occurs during the downward motion only. The duration of tip resistance is about one-quarter of the period of excitation. Also, tip resistance does not depend directly on the static surcharge. Acceleration must necessarily be measured in order to compute pile tip resistance. An important soil property, the dynamic bulk modulus, may be predicted from the theory. The theory is also applicable to impactive pile driving practice provided actual hammer force and acceleration history are known. For reasons of economy and damage to piles, the input force level should not exceed the optimum value prescribed earlier.

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