Zero Extension Line Displacement Theory of Earth Pressure

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Abstract: The simple zero extension line theory (ZEL) proposed by authors [1986] to predict dynamic passive pressure is used to evaluate the active and passive earth pressure coefficients as a function of incremental translation and rotation of an element of wall at a given depth moving away or towards a dense sand backfill.

The general expression for the pressure at depth h of wall with rotation θ and translation Δ is similar to classical earth pressure theory:

\[ P_P = k_{p,y} y + k_{p,q} q \]  
\[ P_A = k_{a,y} y + k_{a,q} q \]  

Where \( k_{p,y} \) and \( k_{a,y} \) are the passive pressure coefficients due to gravity (unit weight) \( γ \) and surcharge \( q \) and \( k_{p,q} \) and \( k_{a,q} \) are active pressure coefficients. It is shown that the four earth pressure coefficients are functions of the angle of dilation of dense and \( v \) and relative translation \( Δ/h \) and rotation \( θ \).

Formulas and charts are shown and comparison with experimental results reported in the literature are made.

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Introduction: The zero extension line concept (line with linear strain equal to zero) was first introduced by Roscoe (1970) for interpretation of the strain field in a sand wedge. The simple zero extension line field was proposed by James and Bramley (1973, 1971) to interpret the result of the sand strain pattern for retaining walls for the passive case. Bramley and Milligan (1975), Milligan and Bramley (1976) and Milligan (1985) further used the zero extension line concept for combined active and passive pressures for retaining walls and sheetpiles. Habibagahi (1975) evaluated earth pressure coefficients by using simple zero extension line theory to predict stresses on a retaining wall. The authors (1980) extended the theory to predict dynamic passive pressure coefficients. Behzadpour (1987) evaluated bearing capacity, and Jahanshish (1983, 1989) presented passive pressure coefficients for a dense sand with a constant angle of cohesion for motion of a wall above the bottom, top and translation of the wall into a dense sand.

In this work the formulation is extended to active earth pressure prediction. It is shown that results of the motion of a rigid wall about the top or translation is not accurately predicted by ZEL theory.

The earth pressure coefficients are evaluated for an element of a wall at depth h with displacement components consisting of translation Δ and rotation θ. Graphs and formulations are presented which show that the coefficients are basically functions of Δ/h and θ. Comparison is also made with experimental results which have been published in the literature.

These formulations make it possible to predict pressure on a wall (rigid or flexible) once the relative translation Δ/h and rotation θ are given as a function of depth. The specific information needed for such a prediction are roughness of the wall, angle of
dilation of dense mud, $v$, and angle of internal friction, $\phi$, of the sand as a function of shear strain parameters which are usually evaluated for each pressure level.

Three cases for the passive case consider the wall shown in Fig. 1 with the simple zero extension line not shown. The active case will be presented later in this paper.

The net consists of three zones: Rankine (OC-D), Coulomb (OB-C), and mixed zone (DA-B). The Rankine and mixed zones are composed of straight lines and the Coulomb zone is composed of radial lines and logarithmic spirals. Because the zero extension line theory implies zero linear strain, during the motion of elements of the wall, the zero extension line elements act like a single link. The whole net then displaces like a set of static links connected at nodes. The angle of the net at each node is $\sin v = \sin \nu = \text{beam stress strain ratio}$.

\[ \sin v = \text{beam stress strain ratio} \]  \hspace{1cm} (3)

and $v$ is the angle of dilation.

The angle $\beta_w$ (angle of mixed zone) is computed by the smoothness of the wall and is not explicitly shown in the passive case, where the wall becomes a zero extension line, $\beta_w = 0$ (mixed zone is eliminated) and for a smooth wall $\beta_w = 45 - \nu$ and the Coulomb zone is eliminated. Thus if an element of the wall is depth $b$, incrementally transverses $\Delta$ and remains $\Delta$. A band of zero extension line from the wall is in the surface is activated such that displacements are given by:

\[ U_{bl} = \Delta \tan \nu \]  \hspace{1cm} (4)

\[ U_{b2} = (\Delta \cos \beta_w)(b - \Delta) \tan \nu \]  \hspace{1cm} (5)

\[ U_{b1} = (\Delta \cos \beta_w) b \tan \nu \]  \hspace{1cm} (6)

The interpretation of the above formula is thin for a consequent of an element of wall at point A, the strip of zero extension line ABCD moves such that AB remains, BC
of radial zone equals $\alpha_0$ and $CD$ of Rankine zone also extends. All these
displacements are normal to the radial zero extension line system and make an angle $v$
with strip ABCD during displacement of each joint on this strip. There is no discontinuity
in the displacement field along this strip. Another important result is that the movements
of other strips do not affect this strip for the simple zero extension line.

The stress states in this strip are then:

in the Rankine zone along CD:

$$\tau_{xy} = \alpha_0 \cos \beta_y \cos (\beta_x - v)$$

(7)

in the Coulomb zone along BC:

$$\tau_{xy} = (\alpha + \alpha_0) \cos \beta_x \cos (\beta_x - v)$$

(8)

and in the mixed zone along AD:

$$\tau_{xy} = \alpha_0 \cos \beta_y \cos (\beta_y - v)$$

(9)

The stress prisms for an element of zero extension are shown in Fig. 2.

The traction $t$ on the zero extension line $LM$ makes an angle $\delta_1$ with the direction
of the other zero extension line $LM$. This angle is called the developed angle of traction.
If this angle is added with $v$, then the angle of traction normal to the zero extension line is
found. It was proved by the authors (1948) that

$$\tan \delta_1 = \tan \theta - \tan \nu \cos v$$

(10)

As shown by Cole (1967), for a dense sand, during the major part of a shear test
the angle of dilatation remains constant. One such result is presented in Fig. 3.

Actual results of shaking wall model tests carried out by James and Bramley
(1971) presented in Fig. 4, also show that for a shaking wall displacing into a dense sand
the angle of dilatation remains almost a constant.
Fig. 2. The stress pattern for an element of zero extension line.
Fig. 3a Stress-strain

40%
Fig. 3b  Volumetric Strain (Cole, 1967)
Fig. 4  Volumetric strain shear strain for model wall test (Brunsby, 1972)
Thus for wall movement into dense sand one can assume that the simple zero extension line remains unchanged due to the constancy of $v$ during shear of dense sand.

**Passive Pressure Coefficients:** The assumed simple zero extension line with constant $v$ during the shear of sand can be used to predict pressure at the wall. The results are presented in Ghabraimal and Clemente (1981) and Jahandoust (1989). An increment of wall at depth $h$ which moves with relative translation $\Delta h$ and rotation $\theta$, gives shear strains at the strip of zero extension line according to formulas 7, 8, 9.

The results from a simple shear test can be used to evaluate the angle of internal friction corresponding to shear strains. Equilibrium analysis of slices along the zero extension lines can be used to evaluate the traction on the wall. The resulting passive pressure coefficients $k_{pT}$ and $k_{pQ}$ are:

$$k_{pT} = (\cos \delta_w/E) [C + \sin \theta_0]$$

$$k_{pQ} = (\cos \delta_w/E) [D + \sin \theta_0]$$

where

$$\delta_w = \theta_0^{-1} (\cos (2\theta_w - v) \sin \theta_0 + \sin (2\theta_w - v) \sin \theta_0 + v)]$$

$$k = \tan (2\theta_0 + v) + \tan v$$

Eqs. 1, 11, 12, 13, 14, 15, 16, 17, 18, 19 are shown in Fig. 1.

$$\theta_0 = \pi/2 - v/2 - \delta_w/2 + 1/2 \sin (\sin \theta_0 \sin \theta_0)$$

$$\delta_w = n/2 - l_2 + l_1 + v + (3\theta_w - \theta_0)$$

$$h = \theta_0 \lambda = \tan (2\theta_0 + v) + 2 \tan v$$
\[
A = \frac{\cos(\delta_0^s + \phi_0^s) \cos(\delta_0^d + \phi_0^d) \sin(\delta_0^d + \phi_0^d)}{\sin \beta_0 \cos(2\delta_0^d + \phi_0^d)}
\]
\[
B = \frac{\cos(\delta_0^d + \phi_0^d) \cos(\delta_0^d + \phi_0^d) \cos \lambda}{\cos(\delta_0^d + \phi_0^d) \cos(\delta_0^d + \phi_0^d) \cos \lambda}
\]
\[
C = \frac{\cos \lambda \sin(\delta_0^d + \phi_0^d)}{\cos(2\delta_0^d + \phi_0^d)}
\]
\[
D = \frac{\cos \lambda \sin(\delta_0^d + \phi_0^d)}{\sin \beta_0 \cos(2\delta_0^d + \phi_0^d)}
\]
\[
E = \frac{\cos \lambda \cos(\delta_0^d + \phi_0^d)}{\cos(2\delta_0^d + \phi_0^d)}
\]

Active pressure coefficients: The simple zero extension line for the same case is shown in Fig. 5.

The direction of the minor principal stress is now close to the surface, therefore, formulae 11 to 25 will be used and the active pressure coefficients \( \lambda, \beta \) due to gravity and surface effect will be evaluated. The results indicate that formulae 11 to 25 can be used if the signs of \( \delta_0^d, \delta_0^d, \delta_0^d, \delta_0^d, \phi, \lambda, \beta, \) and \( \lambda^* \) are changed. This is similar to the work of Shibagaki (1979) who studied the same phenomenon.
Fig. 5  Zero extension line not for the active case.
Criterions for selection of $\beta_w$: The expressions for active and passive coefficients depend on $\beta_w$, which is the angle of the mixed zone with the wall. An element of the wall is shown in Fig. 6 for the passive case.

The pole $p$ can move between points D and A and there is no ambiguity for selection of $\beta_w$ and using formula (13) to obtain $\delta_w$. However, for the active case shown in Fig. 7 there are two positions N and M with the same $\delta_w$.

This gives rise to two positions for the pole, P1 and P2, which yields two values of $\beta_w$ shown by $\beta_{w1}$ and $\beta_{w2}$. Thus, like other limit equilibrium methods, the active pressure coefficient for both values has to be calculated and the larger one chosen. The solution indicates that the correct choice is $\beta_{w1}$. This gives an interesting result, i.e., unlike the passive case for the active case even for a completely rough wall $\delta_w = \delta_{wM} + \frac{\nu}{k} \delta_w$ does not become zero. In fact, it becomes $\beta_{w1} = \beta_{wM}$ indicating that the wall cannot become a zero extension line even in the case of a completely rough wall. For the smooth wall, the value of $\beta_{w1}$ is $45 - \frac{\nu}{k}$. Thus the angle $\beta_{w1}$ can change very little and at a rough the rough wall in an active case behaves very similar to the smooth wall. The experimental results of James and Baushby substantiate this conclusion.

Calculation Process: The normal calculation process is as follows: From the results of a simple shear test for dense sand, assume a constant $\nu$ to be used in calculations. Then having wall rotated $\theta$ and relative settlement $\Delta_h$ and depth $h$, calculate the shear modulus in the mixed Gouraud and Rankine zones. Also, from the results of the simple shear test (variation of the $\phi$ versus shear strain) calculate the angle of internal friction developed in the three zones and the enveloped angles of traction on zero extension lines ($\delta_{w1}, \delta_{wM}, \delta_{w2}$). The passive pressure coefficients ($\kappa_{p1}$ and $\kappa_{p2}$) of the active pressure.
coefficients ($k_x$ and $k_y$) can be calculated and the corresponding pressure on the element of the wall at depth $h$. For the general wall movement either flexible or rigid calculation pressures at a sufficient number of points to develop a pressure curve on the wall and the total passive or active force.

**Selection of Shear Test Results for Active Case:** Considering that the passive case simulates loading conditions on an element of the soil and the active case simulates the unloading condition on an element of the wall, it can be concluded that the results of shear tests which are simulating the loading condition cannot be used to predict active behavior. The results of the shear test shown in Fig. 3 peaks at 0.08 shear strain. This shear strain is at least 20 times larger than the rotation in conditions required to achieve the minimum active pressure coefficients.

A review of literature was undertaken to compare test results for loading and unloading similar to the passive and active cases. Sample results are presented here. In the 1980 conference on shear strain relations of soil, Lade (1980) several tests starting from an all round pressure of 68.9 kPa (10 psf) were conducted. The lateral stress was increased (passive case) and in a similar test, the lateral stress is decreased (similar to active case). The data from the tests were reduced to plot $\sin \theta$ vs. shear strain shown in Fig. 7a. In the Greensboro Conference, Gokhale, Levine and Vardoulakis (1982) tests #423 and #424 are similar to the passive and active cases. These data were reduced to plot $\sin \theta$ vs. shear strain shown in Fig. 7b. Personal communication with Professor Paul Lade of the University of California at Los Angeles (May, 1988) provided results from plane strain tests similar to the active and passive cases. The results of Test 5069 are presented in Fig. 7c and 7d. The test DE simulates unloading and test EFG simulates loading. All of
Fig. 7a  Triaxial Test Results on Sand, Lade (1910)
Fig. 7b  Laboratory Test Results on Sand (Test No. 421 and 423), Gobieus et al. (1982)
Fig. 7c  Laboratory Plane Strain Test Results on Sand (Test F305) Labe (1989)
Fig. 74  Laboratory Plate Shear Test Results on Sand (Test F005) Lade (1989)

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1)
these tests indicate that although the peak of σn is σ for the active and passive cases is nearly equal, the shear strain for the passive peak is from 3 to 8 times larger than the shear strain for the active peak of σn. The authors recommend the use of shear test results such as Fig. 3 for the passive case and use of the same curve but a reduced γ on the order of 1/5 for the active case.

Another: The shear test results selected for a dense sand for the passive and active pressure calculations is from Cole (1967) as shown in Fig. 5. This gives a σ = 15° and an initial σ = 25° at void ratio of 0.554. This test was chosen because extensive reaming wall model tests by James and Braasch (1970) were run on a similar sand. For the active case the shear strain axis was reduced to a scale of 1/5. The backfill surface is horizontal, U = 0, and the wall is vertical. The passive pressure coefficients are shown on Fig. 8 and Fig. 9 for the gravity effect and surcharge coefficients for σ = 5°. It is clear that the choice of the angle of dilation has very little effect on the passive pressure coefficients. Figs. 12 and 13 show the passive pressure coefficients for a smooth wall with σ = 15°.

For the active pressure coefficients rough and smooth walls behave very similar to each other. The result of active pressure coefficients for a smooth wall is shown in Fig. 15. It should be noted that for smooth walls the effect of surcharge and gravity are the same (σn = kσ, and kσ = kσ), similar to classical theory.

It is noted on the passive case, the coefficients increase first due to shearing and then decrease similar to the results of the simple shear test. In the active case, the coefficients decrease first and then increase. For the case of a smooth wall, since
Fig. 9 Passive pressure coefficient (gravity, rough wall, $v = 15$)
Fig. 9 Passive pressure coefficient (surcharge, rough wall, $v = 15$)

870
Fig. 10 Passive pressure coefficient (gravity, rough wall, v = 5)
Fig. 11 Passive pressure coefficient (nutshale, rough wall, $v = 5$)
Fig. 11 Passive pressure coefficients (gravity, smooth wall, $v = 33$)
Fig. 13  Passive pressure coefficient (enough, smooth wall, $v = 15$)
Fig. 14  Active pressure coefficient (gravity, smooth wall, v = 15)
Fig. 15  Active pressure coefficient (or change, smooth wall, \( \nu = 15 \))
Fig. 16 Passive pressure coefficient for rough wall resting about the bottom (a) gravity (b) surcharge effect
Fig. 17 Passive pressure coefficients for smooth wall translating into dense sand (gravity effect)
Fig. 18  Active pressure coefficient for smooth wall rotating about the bottom
translational of the base does not produce shear, the translation has not effect and the
dominant component is the rotation of the element of the wall.

Comparison with Experimental Results: The results obtained from the zero extension
line theory have been compared with tests for a rigid wall translation, or rotation about the
bottom. Because the movement of the wall is known with respect to depth at each stage
of wall movement, the pressure of the wall can be calculated as a function of depth and
the overall earth pressure coefficient can then be calculated. It should be noted that the
zero extension line theory is not suitable for rotation about the top and is not applicable
for translation of smooth walls as noted by James and Brambley (1971). Passive pressure
coefficients were calculated by Jambadaiv (1988) and are presented in Fig. 16 for a rough
wall. Also shown are the results of James and Brambley (1970). For a smooth wall, in the
passive case, the results are presented in Fig. 17 and the experimental results of Brambley
(1972) are also shown on the figure.

For the active case of a rotating wall moving about the bottom the active pressure
coefficients are presented in Fig. 18. The field test results by Matsuo, Kenosh and Yagi
(1975) on a 10 m taper rotating wall are also shown on the figure. The test results of Fang
and Iwasho (1996) are also shown on the figure.

It is clear that although the theory predicts the general trend of the experimental
results, the best match is for the passive case with rotation about the bottom. For the
active case and rotation about the bottom, the experimental results decrease more rapidly
than the theory. It should be again noted that the shear test results for the active case
have used the same sin φ scale, but have used a 1/5 reduction in shear stress, considering
the active case to be unloading.
Conclusions: Based on the above results and discussion, the following conclusions can be made:

1. The simple zero extension line theory (ZEL) is capable of predicting earth pressure coefficients due to an effect of gravity and surcharge in terms of position and relative translation of an element of wall at a given depth for dense sand.

2. The theory can be used to calculate earth pressure once the movement of the wall is given as a fraction of depth. The best comparison with experimental results is found for the passive case with tension about the bottom.

3. The theory predicts similar behavior for rough and smooth walls for the active case as shown by experimental results. The relative movement of the walls are correctly simulated if the shear test results are used with the same soil sample but a reduction of 1/3 scale for shear stress considering the active case to be unloading.
References


Lade, P.V. (1989), Personal Communication


