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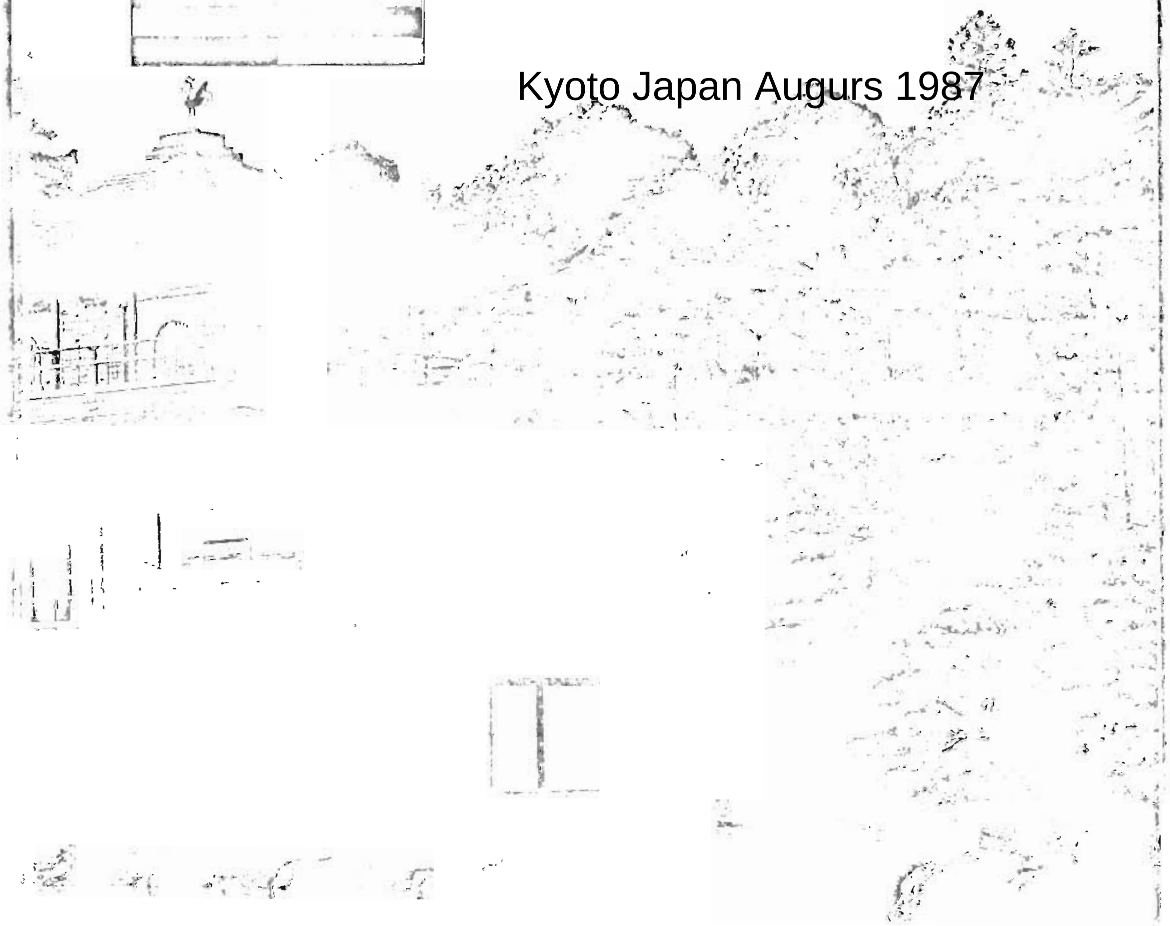

Zero Extension Line Theory of Static and Dynamic Bearing Capacity

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SYNOPSIS

A simple zero extension line field is used to calculate static and dynamic bearing capacity coefficients for sandy soils. Equations and charts for computing these coefficients are presented. The coefficients depend on the angle of friction of footing and angle of friction of sand. The charts also enable evaluation of static and dynamic bearing capacity factors, given angle of dilation of sand and roughness coefficient of the footing. The predicted values compare favorably with sokolovski, except for surcharge coefficient.

INTRODUCTION

The simple zero extension line field first proposed by Roscoe (70) has been used by several authors to explain pattern of strain and pattern of stresses behind retaining walls for static and dynamic problems (Bransby, 75; James, 70; Wroth, 76).

In this work the static and dynamic bearing capacity for sandy soils has been evaluated using this simple zero extension line field. A brief review of zero extension line and induced traction on this line as well as constitutive relations for the sand is first presented.

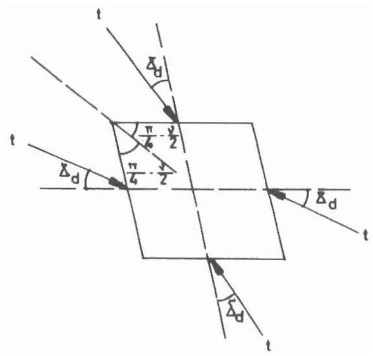


Fig. (1)

REVIEW

When an element of sand undergoes incremental deformation, there are two directions along which linear strain is zero. These two directions are called zero extensions line directions; they make an angle equal to $\frac{\pi}{4} - \frac{\nu}{2}$ with the direction of major principal compressive stresses, where ν is the angle of dilation of sand and is defined as

$$\sin\nu = \frac{dv/v}{d\,\gamma_{max}} \qquad (1)$$

in which dv/v is the volumetric strain and γ_{max} is the maximum angular shearing strain.

Fig. (1) shows a quadrilateral of zero extension line; ν is positive for dense sand and negative for loose sand. Angle δ_d, the developed angle of traction t with the other zero extension line direction, is evaluated for dense sand using Row's Theory and for loose sand using theory proposed by GHAHRAMANI (79). Thus for dense sand

$$\tan\delta_d = \frac{\cos\nu\,\tan\delta}{1 + \sin\nu\,\tan\delta} \qquad (2)$$

$$\sin\phi = \frac{\sin\nu\,\tan\delta}{1 + \sin\nu\,\tan\delta} \qquad (3)$$

and for loose sand

$$\delta_d = \delta \qquad (4)$$

$$\sin\phi = \cos\nu\,\tan\delta + \sin\nu \qquad (5)$$

where ϕ is the angle of internal friction and δ, which is the δ_d at $\nu = 0$, is assumed to be 30 degrees. It should be noted that because

$$\sin\phi = \frac{\sin(\delta_d + \nu)}{\cos\delta_d} \qquad (6)$$

any relation between ϕ and ν can give us δ_d and the angle that traction makes on the zero extension line can be evaluated.

The simple zero extension line field is shown on Fig. (2). It is composed of a Rankine zone OAB, a Goursat logarithmic spiral zone OAC and a mixed zone OCD. If it is assumed that the extereme zero extension line DCAB is motionless, then on any line, the acceleration if starts from zero velocity is distributed as shown in Fig. (3)

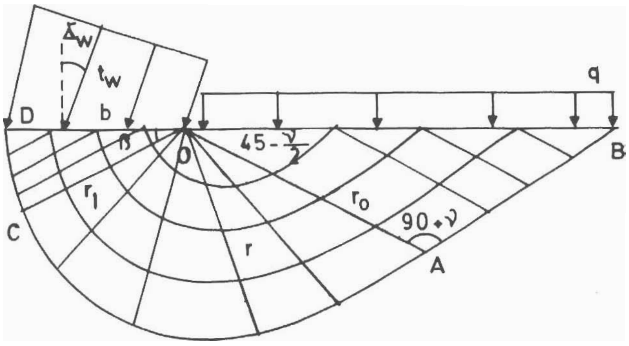


Fig. (2)

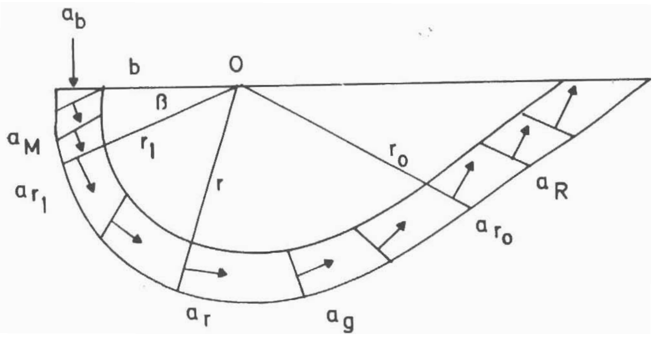


Fig. (3)

In Mixed zone we have

$$a_m = \frac{a_b}{\cos\beta} \tag{7}$$

In Goursat zone

$$a_g = \frac{r}{r_1}\frac{a_b}{\cos\beta} \tag{8}$$

and in Rankine zone

$$a_R = \frac{r_0}{r_1}\frac{a_b}{\cos\beta} \tag{9}$$

where a_b is vertical acceleration of footing at distance b; and a_m is acceleration in mixed zone, a_g, acceleration in Goursat zone, and a_R is acceleration in Rankine zone; angle β is related to the roughness of footing

$$\beta = \frac{\pi}{4} + \frac{\nu}{2} - \frac{\delta_w}{2} - \tfrac{1}{2}\sin^{-1}\left(\frac{\sin\delta_w \cos\delta_d}{\sin(\delta_d + \nu)}\right) \tag{10}$$

δ_w is the footing friction angle with soil: for a complete rough footing $\beta = 0$ and $\delta_w = \delta_d + \nu$ and for smooth footing $\beta = \frac{\pi}{4} + \frac{\nu}{2}$ and $\delta_w = 0$. A roughness coefficient can also be defined as $C_r = \frac{\tan\delta_w}{\tan(\delta_d+\nu)}$ which is one for completely rough footing and zero for smooth footing.

STRESSES IN RANKINE STRIP

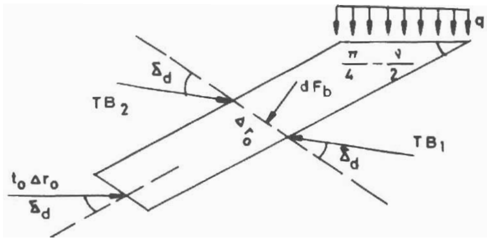


Fig. (4)

The forces on a Rankine strip are shown in Fig. (4) t_o is the traction acting at r_0. TB_2, TB_1 are forces at top and bottom and q is surcharge at the surface of footing; it is seen that because TB_2 and TB_1 are parallel, resolution of forces normal to this direction enables evaluation for t_o in term of q and the body forces dF_b which can be either inertial term for dynamic case or weight for static case. After algebraic manipulation one obtains,

$$t_o \cos(2\delta_d + \nu) = f_o + q\,C \tag{11}$$

where

$$C = 2q\cos\left(\frac{\pi}{4} - \frac{\nu}{2}\right)\sin\left(\frac{\pi}{4} + \frac{\nu}{2} + \delta_d\right) \tag{12}$$

and

$$f_o = \int \frac{dF_t}{\Delta r_o} \tag{13}$$

where dF_t is the projection of inertial force dF_b on the direction normal to TB_1 or TB_2.

STRESSES IN MIXED ZONE

If t_w is the traction under the footing making δ_w with vertical and if t_1 is the traction at r_1 at the edge of mixed zone, TB_1 and TB_2 are forces on the sides of the Rankine strip and dF_b body force element on this strip, then projecting on direction normal to $TB_1 - TB_2$ one gets

$$t_w \cos(\beta + \delta_w + \delta_d) = t_1 \cos(2\delta_d + \nu)\frac{\cos(\beta-\nu)}{\cos\nu} + f_2 \tag{14}$$

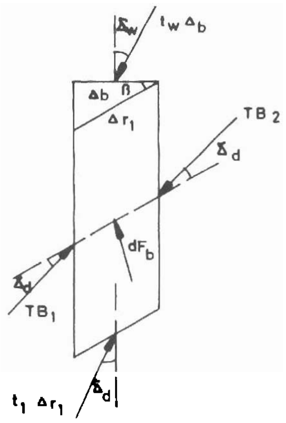


Fig. (5)

where

$$f_2 = \int \frac{dF_t}{\Delta b} \qquad (15)$$

where dF_t is the projection of dF_b on direction normal to TB_1 or TB_2. The above relation enables evaluation of traction under the footing in terms of traction at the edge of mixed zone t_1 and body forces.

STRESSES IN GOURSAT ZONE

Algebraic manipulation similar to GHAHRAMANI and C. lemence (80) gives us the following relation

$$\cos(\nu + 2\delta_d)t_o\, A - \cos(\nu + 2\delta_d)t_1\, B + f_1 = 0 \qquad (16)$$

where t_o and t_1 are traction stresses at both sides r and r_1 of Goursat zone.

$$A = e^{[\tan(\nu + 2\delta_d) + \tan\nu]\beta_1} \qquad (17)$$

$$B = e^{[\tan(\nu + 2\delta_d) + \tan\nu]\beta} \qquad (18)$$

$$\beta_1 = \frac{3\pi}{4} + \frac{\nu}{2} \qquad (19)$$

and

$$f_1 = \int_{\beta}^{\beta_1} \frac{dF_t}{\Delta r}\, e^{[\tan(\nu + 2\delta_d) + \tan\nu]\theta} \qquad (20)$$

where dF_t is the projection normal to TB_1 and TB_2 of body forces dF_b and θ is the angle the element line r makes with footing. See Fig. (6)

PRESSURE UNDER THE FOOTING

Recognizing that the p pressure under the footing is related to traction t_w by the following relation,

$$p = t_w \cos\delta_w \qquad (21)$$

and using the stress relation in Rankine, Goursat and Mixed zones we get:

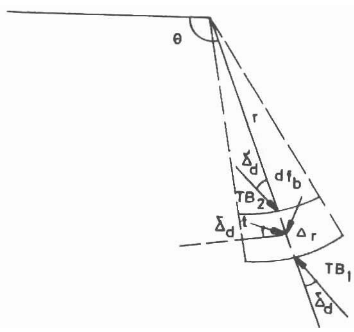


Fig. (6)

$$p = \frac{\cos(\beta - \nu)\cos\delta_w}{B\cos\nu\cos(\beta + \delta_w + \delta_d)}[f_1 + Af_o + ACq]$$

$$+ \frac{\cos\delta_w}{\cos(\beta + \delta_w + \delta_d)} f_2 \qquad (22)$$

Thus pressure can be evaluated in terms of surcharge and body forces.

STATIC CASE

For the static case the body force

$$dF_b = \gamma dA \qquad (23)$$

where dA is element of area; evaluating f_o, f_1 and f_2 and substiting in p we get

$$f_o = \gamma b\cos(\beta - \nu)\, e^{\tan\nu(\beta_1 - \beta)} \sin(\frac{\pi}{4} + \frac{\nu}{2} + \delta_d) \qquad (24)$$

$$f_1 = \frac{\gamma b}{\cos\nu}\cos(\beta-\nu)\, e^{-\beta\tan\nu}\,[-\sin(\lambda+\beta_1+\delta_d)e^{\beta_1\tan\lambda}$$

$$+ \sin(\lambda + \beta + \delta_d)\, e^{\beta\tan\lambda}] \qquad (25)$$

where

$$\tan\lambda = \tan(\nu + 2\delta_d) + 2\tan\nu \qquad (26)$$

$$f_2 = -\frac{\gamma b}{\cos\nu}\cos(\beta - \nu)\cos(\delta_d + \beta) \qquad (27)$$

and

$$p = N_q\, q + N_{\gamma_{static}}\, \gamma\, b \qquad (28)$$

where

$$N_q = \frac{\cos(\beta-\nu)\cos\delta_w}{\cos\nu\cos(\beta+\delta_w+\delta_d)}\, e^{[\tan(\nu + 2\delta_d) + \tan\nu](\beta_1-\beta)}$$

$$\times 2\cos(\frac{\pi}{4} - \frac{\nu}{2})\sin(\frac{\pi}{4} + \frac{\nu}{2} + \delta_d) \qquad (29)$$

$$N_{\gamma_{static}} = \frac{\cos^2(\beta-\nu)\cos\delta_w}{\cos^2\beta\cos(\delta_w+\delta_d+\beta)} \{- \frac{\sin\beta\cos(\beta+\delta_d)\cos\nu}{\cos(\beta-\nu)}$$

$$+ \cos\lambda \sin(\lambda + \delta_d + \beta)$$

$$+ e^{(\beta_1-\beta)\tan\lambda} [\cos\nu \sin(\frac{\pi}{4} + \frac{\nu}{2} + \delta_d)$$

$$- \cos\lambda \sin(\lambda + \delta_d + \beta_1)]\} \quad (30)$$

DYNAMIC CASE

When the footing is accelerated in sand with vertical acceleration a_b at distance b from the edge of the footing, the inertial term

$$dF_b = \rho\ a\ dA \quad (31)$$

where a is acceleration which can be obtained by the geometrical and acceleration relation developed earlier and dA is area element and ρ is soil density. Now the f_o, f_1, and f_2 is evaluated giving

$$f_o = \rho\ a_b\ b\ e^{2\tan\nu(\beta_1-\beta)} \frac{\cos(\beta-\nu)\cos\delta_d}{\cos\beta} \quad (32)$$

$$f_1 = \rho\ a_b\ b \frac{\cos(\beta-\nu)}{\cos\beta\cos\nu} \cos\delta_d \frac{e^{-2\tan\nu\beta}}{3\tan\nu+\tan(\nu+2\delta_d)}$$

$$\times \{e^{[3\tan\nu+\tan(\nu+2\delta_d)]\beta_1} - e^{[3\tan\nu+\tan(\nu+2\delta_d)]\beta}\} \quad (33)$$

$$f_2 = \rho\ a_b\ b \cos\delta_d \frac{\sin\beta}{\cos\nu} \frac{\cos(\beta-\nu)}{\cos\beta} \quad (34)$$

substitution in pressure term yields

$$p_{dynamic} = \rho\ a_b\ b\ N_{\gamma_{dynamic}} \quad (35)$$

where

$$N_{\gamma_{dynamic}} = \frac{\cos\delta_d\cos\delta_w\cos^2(\beta-\nu)}{\cos\beta\cos(\beta+\delta_w+\delta_d)\cos^2\nu} \{\frac{\sin\beta\cos\nu}{\cos(\beta-\nu)}$$

$$- \frac{1}{3\tan\nu + \tan(2\delta_d + \nu)} + \Big[(\cos\nu$$

$$+ \frac{1}{3\tan\nu+\tan(\nu+2\delta_d)}\Big] e^{(\beta_1-\beta)[3\tan\nu+\tan(\nu+2\delta_d)]}\} \quad (36)$$

STATIC PLUS DYNAMIC CASE

The linearity t_w with f_o, f_1, f_2 yields the general pressure developed under the footing p accelerating at distance b with acceleration a_b into sand:

$$p = p_{static} + p_{dynamic} \quad (37)$$

$$p = q\ N_q + \gamma b\ N_{\gamma_{static}} + \rho\ b\ a_b\ N_{\gamma_{dynamic}} \quad (38)$$

Thus if friction angle φ, and footing friction angle δ_w is given, N_q, $N_{\gamma_{dynamic}}$ and $N_{\gamma_{static}}$ can be evaluated. Fig. 7,8 and 9 give the results for N_q, $N_{\gamma_{dynamic}}$ and $N_{\gamma_{static}}$.

It should be noted that the diagrams are so presented that if instead of φ, angle ν is given and if instead of δ_w, roughness coefficient C_r is given, the N_q, $N_{\gamma_{dynamic}}$ and $N_{\gamma_{static}}$ can be easily predicted from the Figures (7), (8) and (9). The ratio of $N_{\gamma_{dynamic}}$ to $N_{\gamma_{static}}$ is presented in Fig. (10).

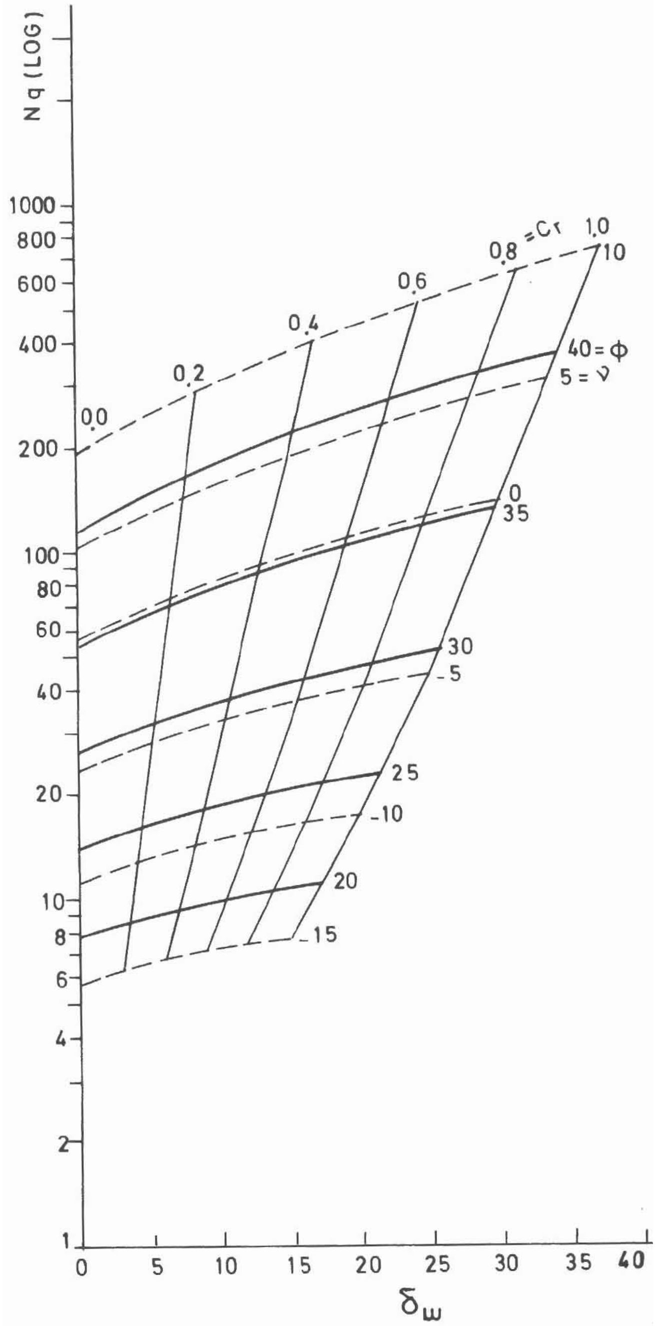


Fig. (7)

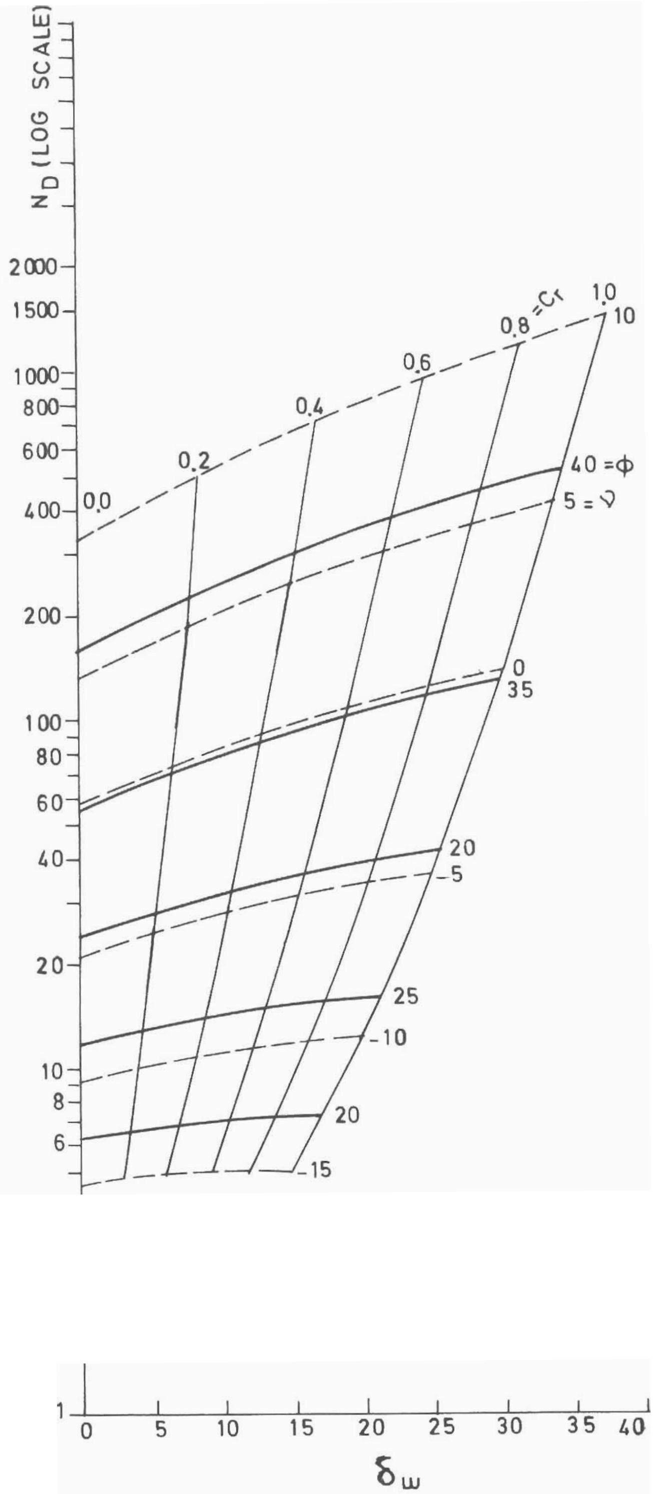
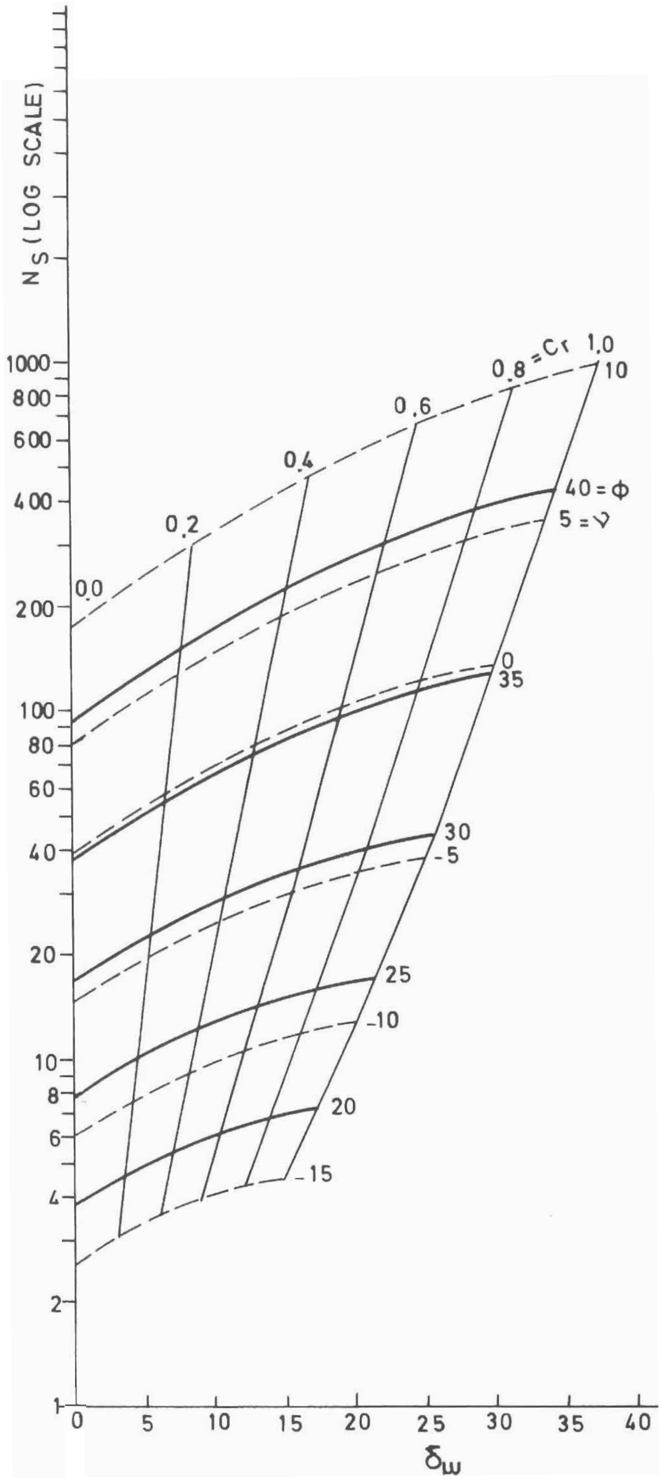

N_D (LOG SCALE)
2000
1500
1000
800
600
400
200
100
80
60
40
20
10
8
6
1
0.0
0.2
0.4
0.6
0.8 = Cr
1.0
10
40 = Φ
5 = ν
0
35
20
-5
25
-10
20
-15
0 5 10 15 20 25 30 35 40
δ_ω

Fig. (8)


N_S (LOG SCALE)
1000
800
600
400
200
100
80
60
40
20
10
8
6
4
2
1
0.0
0.2
0.4
0.6
0.8 = Cr
1.0
10
40 = Φ
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0
35
30
-5
25
-10
20
-15
0 5 10 15 20 25 30 35 40
δ_ω

Fig. (9)

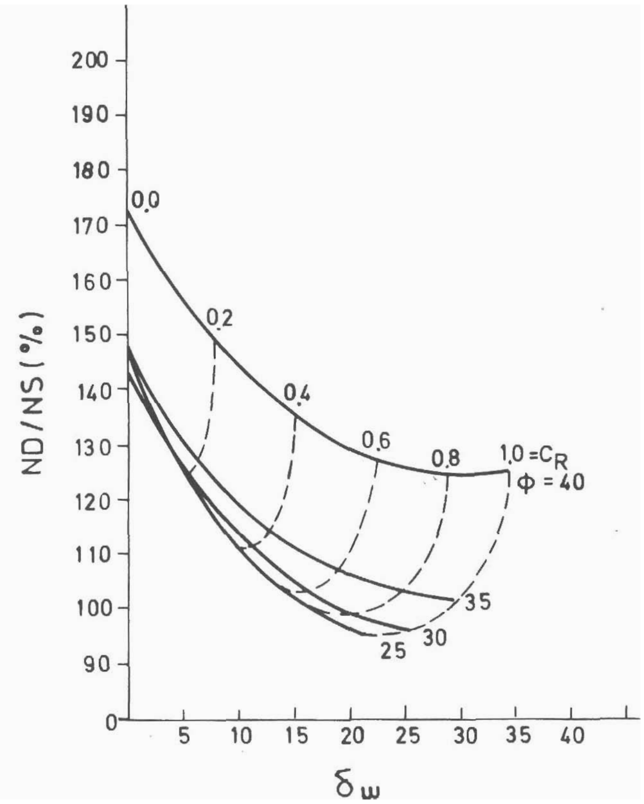

Fig. (10)

DISCUSSION

It is seen that if the footing is accelerated into sand, an added pressure is felt by the footing due to this acceleration. This added dynamic pressure is equal to $N_{\gamma dynamic} \times \rho b a_b$. **It is analogous to an** inertial force. It should be mentioned that if vertical acceleration a_b is uniform then this pressure also varies linearly, however different modes of acceleration of footing yields non linear pressure variation. It is interesting to note that strips of zero extension line field act independent of the condition of neighbouring strips and each resist dynamic motion like an inertial mass.

In comparison with Sokolovski, the $N_{\gamma static}$ values are almost indentical, however the N_q values are larger and sometime even 50% more than the Sokolovski values.

The chart showing the ratio of $N_{\gamma dynamic}$ to $N_{\gamma static}$ can be used to evaluate the dynamic bearing capacity factor for users who prefer to use static bearing capacity factors developed by other theoreis than the simple extension line theory.

CONCLUSIONS

Based on the theoretical development and analysis presented in this paper the following conclusions can be made:

1. **The simple zero extension line field is capable** of predicting static and dynamic bearing capacity factors for sandy soils, for footings having variable roughness.
2. Analogous to the static case, a dynamic bearing capacity factor is evaluated. When this coefficient is multiplied by soil density and the distance from the edge of the footing, the dynamic pressure can be evaluated. **The total pressure** felt by the accelerating footing is the sum of static and this dynamic pressure.
3. The $N_{\gamma static}$ values compare favorably with Sokolovski, however N_q values are generally higher.

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